Pleasechecktheexaminationdetailsbelowbeforeenteringyourcandidateinformation

| Candidatesurname | Othernames |
|---|---|
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Centre Number | Candidate Number

# Pearson Edexcel International Advanced Level

**Time** 1 hour 30 minutes

**Paper reference** **WMA14/01**

# Mathematics

## International Advanced Level
## Pure Mathematics P4

**You must have:**
Mathematical Formulae and Statistical Tables (Yellow), calculator

TotalMarks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  *– there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  *– use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

*Turn over* ▶

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**1.** $$f(x) = \frac{5x + 10}{(1 - x)(2 + 3x)}$$

(a) Write $f(x)$ in partial fraction form.

**(3)**

(b) (i) Hence find, in ascending powers of $x$ up to and including the terms in $x^2$, the binomial series expansion of $f(x)$. Give each coefficient as a simplified fraction.

**(5)**

(ii) Find the range of values of $x$ for which this expansion is valid.

**(1)**

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**Question 1 continued**

**Question 1 continued**

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**Question 1 continued**

**(Total for Question 1 is 9 marks)**

**2.** A set of points $P(x, y)$ is defined by the parametric equations

$$x = \frac{t-1}{2t+1} \qquad y = \frac{6}{2t+1} \qquad t \neq -\frac{1}{2}$$

(a) Show that all points $P(x, y)$ lie on a straight line.

**(4)**

(b) Hence or otherwise, find the $x$ coordinate of the point of intersection of this line and the line with equation $y = x + 12$

**(2)**

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**Question 2 continued**

**(Total for Question 2 is 6 marks)**

**3.**

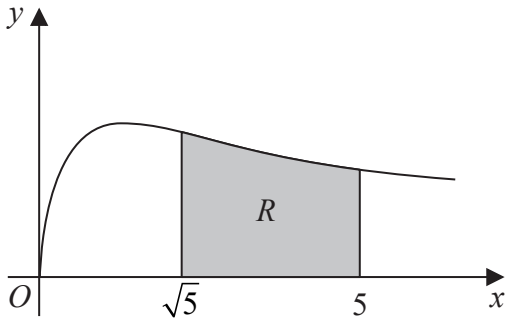



**Figure 1**

**In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

Figure 1 shows a sketch of the curve with equation

$$y = \sqrt{\frac{3x}{3x^2 + 5}} \qquad x \geqslant 0$$

The finite region $R$, shown shaded in Figure 1, is bounded by the curve, the $x$-axis and the lines with equations $x = \sqrt{5}$ and $x = 5$

The region $R$ is rotated through 360° about the $x$-axis.

Use integration to find the exact volume of the solid generated. Give your answer in the form $a \ln b$, where $a$ is an irrational number and $b$ is a prime number.

**(5)**

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**Question 3 continued**

**(Total for Question 3 is 5 marks)**

**4.** (a) Using the substitution $u = \sqrt{2x+1}$, show that

$$\int_4^{12} \sqrt{8x+4}\ \mathrm{e}^{\sqrt{2x+1}}\ \mathrm{d}x$$

may be expressed in the form

$$\int_a^b ku^2\mathrm{e}^u\ \mathrm{d}u$$

where $a$, $b$ and $k$ are constants to be found.

**(4)**

(b) Hence find, by algebraic integration, the exact value of

$$\int_4^{12} \sqrt{8x+4}\ \mathrm{e}^{\sqrt{2x+1}}\ \mathrm{d}x$$

giving your answer in simplest form.

**(5)**

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**Question 4 continued**

**Question 4 continued**

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**Question 4 continued**

**(Total for Question 4 is 9 marks)**

**5.**

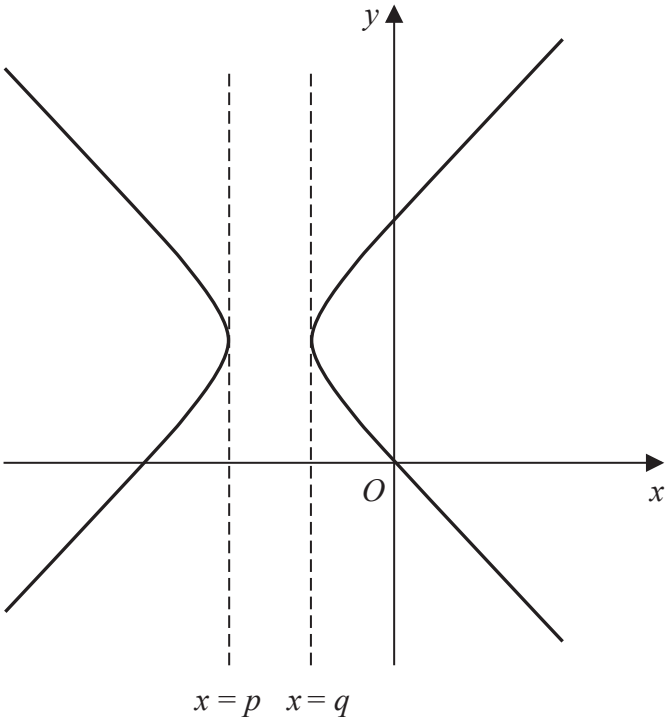



**Figure 2**

Figure 2 shows a sketch of the curve with equation

$$y^2 = 2x^2 + 15x + 10y$$

(a) Find $\frac{dy}{dx}$ in terms of $x$ and $y$.

**(4)**

The curve is not defined for values of $x$ in the interval $(p, q)$, as shown in Figure 2.

(b) Using your answer to part (a) or otherwise, find the value of $p$ and the value of $q$.

(*Solutions relying entirely on calculator technology are not acceptable.*)

**(3)**

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**Question 5 continued**

**Question 5 continued**

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**Question 5 continued**

**(Total for Question 5 is 7 marks)**

**6.** Relative to a fixed origin $O$.

- the point $A$ has position vector $2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$
- the point $B$ has position vector $8\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}$

The line $l$ passes through $A$ and $B$.

(a) (i) Find $\overrightarrow{AB}$

(ii) Find a vector equation for the line $l$

**(3)**

The point $C$ has position vector $3\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$

The point $P$ lies on $l$

Given that $\overrightarrow{CP}$ is perpendicular to $l$

(b) find the position vector of the point $P$

**(5)**

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**Question 6 continued**

**Question 6 continued**

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**Question 6 continued**

**(Total for Question 6 is 8 marks)**

**7.** The volume $V\,\text{cm}^3$ of a spherical balloon with radius $r\,\text{cm}$ is given by the formula

$$V = \frac{4}{3}\pi r^3$$

(a) Find $\dfrac{\mathrm{d}V}{\mathrm{d}r}$ giving your answer in simplest form.

**(1)**

At time $t$ seconds, the volume of the balloon is increasing according to the differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{900}{(2t+3)^2} \qquad t \geqslant 0$$

Given that $V = 0$ when $t = 0$

(b) (i) solve this differential equation to show that

$$V = \frac{300t}{2t+3}$$

(ii) Hence find the upper limit to the volume of the balloon.

**(5)**

(c) Find the radius of the balloon at $t = 3$, giving your answer in cm to 3 significant figures.

**(3)**

(d) Find the rate of increase of the radius of the balloon at $t = 3$, giving your answer to 2 significant figures. Show your working and state the units of your answer.

**(3)**

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**Question 7 continued**

**Question 7 continued**

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**Question 7 continued**

**(Total for Question 7 is 12 marks)**

**8.**

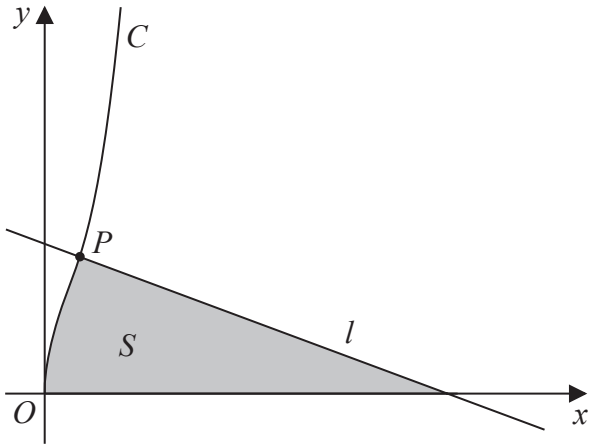



**Figure 3**

**In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

A curve $C$ has parametric equations

$$x = \sin^2 t \qquad y = 2\tan t \qquad 0 \leqslant t < \frac{\pi}{2}$$

The point $P$ with parameter $t = \frac{\pi}{4}$ lies on $C$.

The line $l$ is the normal to $C$ at $P$, as shown in Figure 3.

(a) Show, using calculus, that an equation for $l$ is

$$8y + 2x = 17$$

**(5)**

The region $S$, shown shaded in Figure 3, is bounded by $C$, $l$ and the $x$-axis.

(b) Find, using calculus, the exact area of $S$.

**(6)**

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**Question 8 continued**

**Question 8 continued**

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**Question 8 continued**

**(Total for Question 8 is 11 marks)**

**9.** A student was asked to prove, for $p \in \mathbb{N}$, that

"if $p^3$ is a multiple of 3, then $p$ must be a multiple of 3"

The start of the student's proof by contradiction is shown in the box below.

Assumption:

There exists a number $p$, $p \in \mathbb{N}$, such that $p^3$ is a multiple of 3, and $p$ is NOT a multiple of 3

Let $p = 3k + 1$, $k \in \mathbb{N}$.

Consider $p^3 = (3k + 1)^3 = 27k^3 + 27k^2 + 9k + 1$

$= 3(9k^3 + 9k^2 + 3k) + 1$ which is not a multiple of 3

(a) Show the calculations and statements that are required to complete the proof.

**(3)**

(b) Hence prove, by contradiction, that $\sqrt[3]{3}$ is an irrational number.

**(5)**

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**Question 9 continued**

**Question 9 continued**

**(Total for Question 9 is 8 marks)**

**TOTAL FOR PAPER IS 75 MARKS**

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