GradeMax

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WMA14/01**

Mathematics

International Advanced Level
Pure Mathematics P4

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1. The curve C has equation

$$2x - 4y^2 + 3x^2y = 4x^2 + 8$$

The point $P(3, 2)$ lies on C.

Find the equation of the normal to C at the point P, writing your answer in the form $ax + by + c = 0$ where a, b and c are integers to be found.

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Question 1 continued

Q1

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2. Find the particular solution of the differential equation

$$\frac{dy}{dx} = \frac{4y^2}{\sqrt{4x+5}} \qquad x > -\frac{5}{4}$$

for which $y = \frac{1}{3}$ at $x = -\frac{1}{4}$ giving your answer in the form $y = f(x)$

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Question 2 continued

Q2

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3. $$g(x) = \frac{3x^3 + 8x^2 - 3x - 6}{x(x+3)} \equiv Ax + B + \frac{C}{x} + \frac{D}{x+3}$$

(a) Find the values of the constants A, B, C and D.

(5)

A curve has equation

$$y = g(x) \qquad x > 0$$

Using the answer to part (a),

(b) find $g'(x)$.

(2)

(c) Hence, explain why $g'(x) > 3$ for all values of x in the domain of g.

(1)

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Question 3 continued

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4. $f(x) = \sqrt{1 - 4x^2}$ $\quad |x| < \frac{1}{2}$

(a) Find, in ascending powers of x, the first four non-zero terms of the binomial expansion of $f(x)$. Give each coefficient in simplest form.

(4)

(b) By substituting $x = \frac{1}{4}$ into the binomial expansion of $f(x)$, obtain an approximation for $\sqrt{3}$

Give your answer to 4 decimal places.

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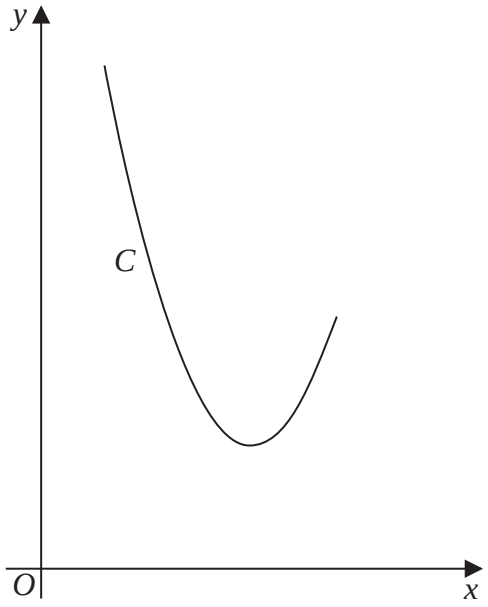


Figure 1

Figure 1 shows a sketch of the curve C with parametric equations

$$x = 5 + 2\tan t \qquad y = 8\sec^2 t \qquad -\frac{\pi}{3} \leqslant t \leqslant \frac{\pi}{4}$$

(a) Use parametric differentiation to find the gradient of C at $x = 3$

(4)

The curve C has equation $y = \mathrm{f}(x)$, where f is a quadratic function.

(b) Find $\mathrm{f}(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants to be found.

(3)

(c) Find the range of f.

(2)

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Question 5 continued

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6. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

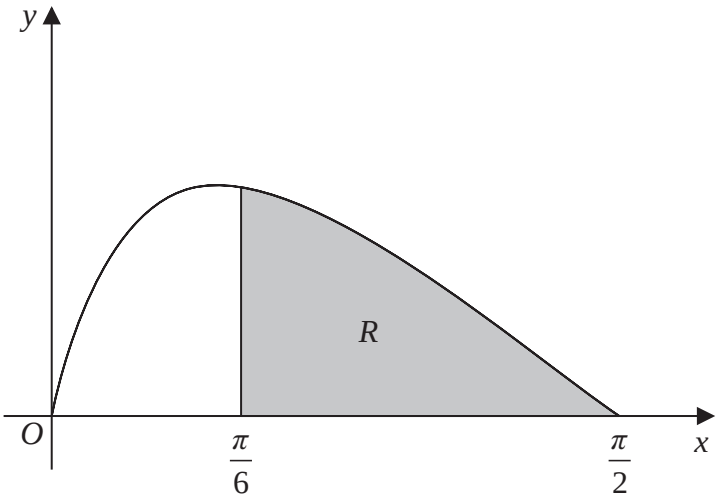


Figure 2

Figure 2 shows a sketch of the curve with equation

$$y = \frac{16\sin 2x}{(3 + 4\sin x)^2} \qquad 0 \leqslant x \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 2, is bounded by the curve, the x-axis and the line with equation $x = \frac{\pi}{6}$

Using the substitution $u = 3 + 4\sin x$, show that the area of R can be written in the form $a + \ln b$, where a and b are rational constants to be found.

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7. With respect to a fixed origin O,

- the line l has equation $\mathbf{r} = \begin{pmatrix} 4 \\ 2 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} -4 \\ -3 \\ 5 \end{pmatrix}$ where λ is a scalar constant
- the point A has position vector $9\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$

Given that X is the point on l nearest to A,

(a) find

(i) the coordinates of X

(ii) the shortest distance from A to l.
Give your answer in the form $\sqrt{d}$, where d is an integer.

(7)

The point B is the image of A after reflection in l.

(b) Find the position vector of B.

(2)

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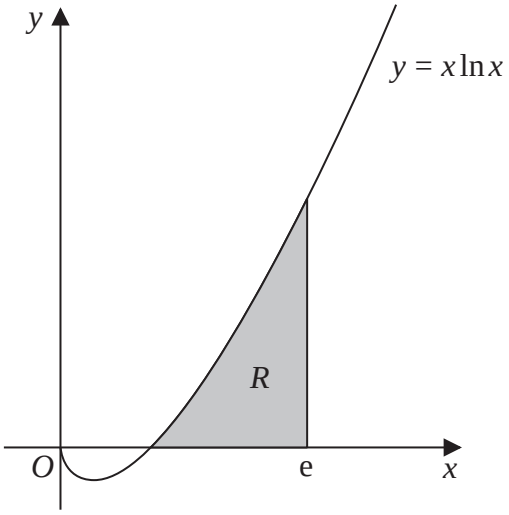


Figure 3

(a) Find $\int x^2 \ln x \, dx$

(3)

Figure 3 shows a sketch of part of the curve with equation

$$y = x \ln x \qquad x > 0$$

The region R, shown shaded in Figure 3, lies entirely above the x-axis and is bounded by the curve, the x-axis and the line with equation $x = e$.

This region is rotated through 2π radians about the x-axis to form a solid of revolution.

(b) Find the exact volume of the solid formed, giving your answer in simplest form.

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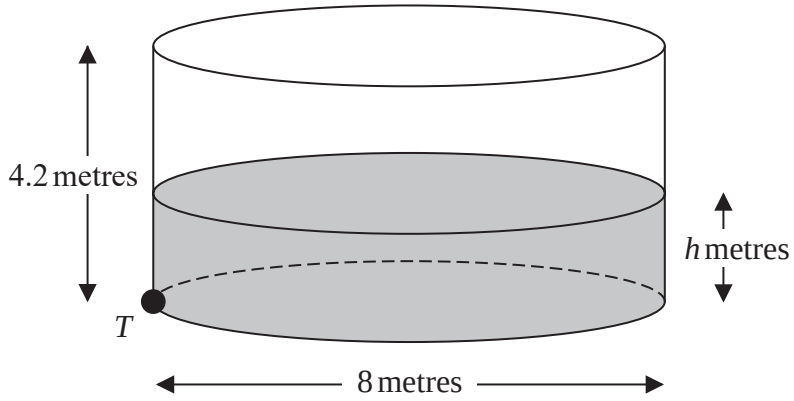


Figure 4

Figure 4 shows a cylindrical tank that contains some water.

The tank has an internal diameter of 8 m and an internal height of 4.2 m.

Water is flowing into the tank at a constant rate of $(0.6\pi)\,\text{m}^3$ per minute.

There is a tap at point T at the bottom of the tank.

At time t minutes after the tap has been opened,

- the depth of the water is h metres
- the water is leaving the tank at a rate of $(0.15\pi h)\,\text{m}^3$ per minute

(a) Show that

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{12 - 3h}{320}$$

(4)

Given that the depth of the water in the tank is 0.5 m when the tap is opened,

(b) find the time taken for the depth of water in the tank to reach 3.5 m.

(6)

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10. (a) A student's attempt to answer the question

"Prove by contradiction that if n^3 is even, then n is even"

is shown below. Line 5 of the proof is missing.

Assume that there exists a number n such that n^3 is even, but n is odd.

If n is odd then $n = 2p + 1$ where $p \in \mathbb{Z}$

So $n^3 = (2p + 1)^3$

$= 8p^3 + 12p^2 + 6p + 1$

$=$

This contradicts our initial assumption, so if n^3 is even, then n is even.

Complete this proof by filling in line 5.

(1)

(b) Hence, prove by contradiction that $\sqrt[3]{2}$ is irrational.

(5)

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TOTAL FOR PAPER: 75 MARKS

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