edexcel

Mark Scheme (Results)

Summer 2015

Pearson Edexcel GCE in Further Pure Mathematics (6667/01)

GradeMax

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2015
Publications Code UA041563
All the material in this publication is copyright
© Pearson Education Ltd 2015

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $*$ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q),\ \text{where} |pq| = |c|$, leading to x = ...

$(ax^2 + bx + c) = (mx + p)(nx + q),\ \text{where} |pq| = |c| \text{ and } |mn| = |a|$, leading to x = ...

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0,\ \ q \neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

June 2015
Further Pure Mathematics FP1 6667
Mark Scheme

Question Number	Scheme	Marks
1.	$(x-5)$ is a factor of f(x) so f(x) = $(x-5)(9x^2...\quad)$	M1
	f(x) = $(x-5)(9x^2+12x+5)$	A1
	Solve $(9x^2+12x+5)=0$ to give $x=$	M1
	$(x=)-\frac{2}{3}-\frac{1}{3}\text{i},\ -\frac{2}{3}+\frac{1}{3}\text{i}$ or $-\frac{2}{3}\pm\frac{1}{3}\text{i}$ or $\frac{-2\pm\text{i}}{3}$ oe (and 5)	A1cao A1ft (5) **(5 marks)**
	Notes M1: Uses (x-5) as factor and begins division or process to obtain quadratic with $9x^2$. Award if no working but quadratic factor completely correct. A1: $9x^2+12x+5$ M1: Solves their quadratic by usual rules leading to $x=$ Award if one complex root correct with no working. Award for $(9x^2+...$ incorrectly factorised to $(3x+p)(3x+q)$, where $\lvert pq\rvert=5$ A1: One correct complex root. Accept any exact equivalent form. Accept single fraction and $\pm$ A1ft: Conjugate of their first complex root.	

Question Number	Scheme	Marks
2. (a)	Let f(x) = $3 + x\sin\left(\frac{x}{4}\right)$ then f (13)= 1.593 [and f(14) = -1.911 need not be seen in (a)]	
	f(13.5) = -0.122, so root in [13, 13.5]	M1 A1
	f(13.25) = 0.746 so root in [13.25, 13.5]	A1 (3)
(b)	$\frac{\alpha-13}{14-\alpha}=\frac{1.593}{1.911}$ or $\frac{\alpha-13}{1}=\frac{1.593}{1.593+1.911}$	M1 A1
	So $\alpha(1.911+1.593)=1.593\times14+13\times1.911$ and $\alpha=\frac{47.145}{3.504}=13.455$	dM1 A1 (4) **(7 marks)**

Notes

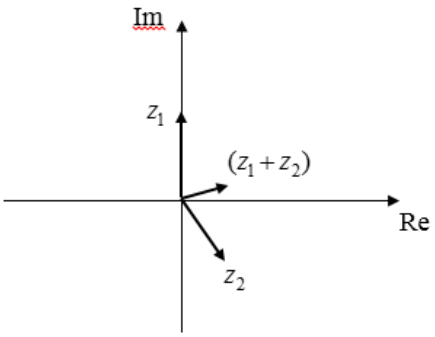
(a)
M1: Evaluate f(13) and f(13.5) giving at least positive, negative **OR** evaluate f(13.5) and f(13.25) to give at least negative, positive. Do not award if using degrees.
A1: f(13.5) = awrt -0.1, f(13.25) = awrt 0.7(5).
A1: Correct interval [13.25, 13.5] or equivalent form with or without boundaries.

(b)
M1: Attempt at linear interpolation on either side of equation with correct signs.
A1: Correct equivalent statement
dM1: Makes alpha subject of formula
A1: cao. Award A0 for 13.456 and 13.454
ALT (b)
Using equation of line
M1: Attempt to find gradient $\frac{y_1-y_0}{x_1-x_0}=\frac{-1.911-1.593}{14-13}(=-3.504...)$, attempt to use $y-y_0=m(x-x_0)$ with either 13 or 14 (gives y = -3.504x + 47.145) and substitute $y=0$
A1: Correct statement after substituting $y=0$ in their equation i.e. 0 = -3.504x + 47.145
dM1: Makes x the subject of the formula
A1: cao. Award A0 for 13.456 and 13.454

Question Number	Scheme	Marks
3. (a)	$\sum_{r=1}^{n}(r+1)(r+4)$	
	$=\sum_{r=1}^{n} r^2+5r+4$	B1
	$=\frac{n}{6}(n+1)(2n+1)+5\frac{n}{2}(n+1)+4n$	M1 A1
	$=\frac{n}{6}\{(n+1)(2n+1)+15(n+1)+24\}$	dM1
	$=\frac{n}{6}\{(2n^2+3n+1)+15n+15+24\}$	
	$=\frac{n}{6}\left(2n^2+18n+40\right)$ or $=\frac{n}{3}\left(n^2+9n+20\right)$	
	$=\frac{n}{3}(n+4)(n+5)$ **** given answer****	A1* (5)
(b)	$\sum_{r=n+1}^{2n}(r+1)(r+4) = \frac{2n}{3}(2n+4)(2n+5)-\frac{n}{3}(n+4)(n+5)$	M1
	$=\frac{n}{3}\{8n^2+36n+40-n^2-9n-20\}$	dM1
	$=\frac{n}{3}\{7n^2+27n+20\}=\frac{n}{3}(n+1)(7n+20)$ or $a=7$, $b=20$	A1 (3)
		(8 marks)

Notes

(a)
B1: Expands bracket correctly to r^2+5r+4
M1: Uses $\frac{n}{6}(n+1)(2n+1)$ or $\frac{n}{2}(n+1)$ correctly.
A1: Completely correct expression.
dM1: Attempts to remove factor $\frac{n}{6}$ or $\frac{n}{3}$ to obtain a quadratic factor. Need not be 3 term.
A1: Completely correct work including a step with a collected **3 term** quadratic prior in the bracket with correct printed answer.
Accept approach which starts with LHS and then RHS which meet at $\frac{n^3}{3}+3n^2+\frac{20n}{3}$. Award marks as above.
NB If induction attempted then typically this may only score the first B1.
However, consider the solution carefully and award as above if seen in the body of the induction attempt.
(b) M1: Uses f(2n) – f(n) or f(2n) – f(n+1) correctly. Require all 3 terms in 2n (and n+1 if used).
dM1: Attempts to remove factor $\frac{n}{6}$ or $\frac{n}{3}$ to obtain a quadratic factor. Need not be 3 term.
A1: Either in expression or as above.

Question Number	Scheme	Marks
4. (a)	$z_2 = \dfrac{6(1-\mathrm{i}\sqrt{3})}{(1+\mathrm{i}\sqrt{3})(1-\mathrm{i}\sqrt{3})} = \dfrac{6(1-\mathrm{i}\sqrt{3})}{4}$	M1
	$z_2 = \dfrac{6(1-\mathrm{i}\sqrt{3})}{4}\left(= \dfrac{3}{2} - \mathrm{i}\dfrac{3}{2}\sqrt{3}\right)$	A1 (2)
(b)	$\lvert z_2\rvert = \sqrt{\left(\dfrac{3}{2}\right)^2 + \left(\dfrac{3\sqrt{3}}{2}\right)^2} = \sqrt{\dfrac{9}{4} + \dfrac{27}{4}}$	M1
	The modulus of z_2 is 3	A1
	$\tan\theta = (\pm)\sqrt{3}$ and attempts to find θ	M1
	and the argument is $-\dfrac{\pi}{3}$	A1 (4)
(c)		M1 A1 (2)
		(8 marks)

Notes

(a) M1: Multiplies numerator and denominator by $1-\mathrm{i}\sqrt{3}$
A1: any correct equivalent with real denominator.
(b) M1: Uses correct method for modulus for their z_2 in part (a)
A1: for 3 only
M1: Uses tan or inverse tan
A1: $-\dfrac{\pi}{3}$ accept $\dfrac{5\pi}{3}$
NB Answers only then award 4/4 but arg must be in terms of π
(c) M1:
Either z_1 on imaginary axis and labelled with z_1 or 3i or (0,3) or axis labelled 3;
or their z_2 in the correct quadrant labelled z_2 or $\dfrac{3}{2}-\mathrm{i}\dfrac{3}{2}\sqrt{3}$ or $\left(\dfrac{3}{2},-\dfrac{3}{2}\sqrt{3}\right)$ or axes labelled
or their $a+b\mathrm{i}$ or their (a,b) or axes labelled.
Axes need not be labelled Re and Im.
A1: All 3 correct ie z_1 on positive imaginary axis, z_2 in 4th quadrant and z_1+z_2 in the first quadrant.
Accept points or lines. Arrows not required.

Question Number	Scheme	Marks
5. (a)	$\frac{dy}{dx}=-\frac{9}{x^2}$ or $\frac{dy}{dx}=-\frac{y}{x}$ or $\frac{dy}{dx}=-\frac{1}{t^2}$	M1
	so gradient at $x=6$ or $t=2$ is $-\frac{9}{36}$ or $-\frac{\frac{3}{2}}{6}$ or $-\frac{1}{4}$ o.e.	A1
	Gradient of normal is $-\frac{1}{m}$ $(=4)$	M1
	Equation of normal is $y-\frac{3}{2}=4(x-6)$	dM1
	So $2y-8x+45=0$ ****given answer****	A1 *
		(5)
(b)	$\frac{18}{x}-8x+45=0$ or $2y-\frac{72}{y}+45=0$ or $x(4x-22.5)=9$ or $y\left(\frac{y}{4}+\frac{45}{8}\right)=9$ o.e.	M1
	$8x^2-45x-18=0$ or $2y^2+45y-72=0$	
	So $x=-\frac{3}{8}$ or $y=-24$	A1
	Finds other ordinate: $\left(-\frac{3}{8},-24\right)$	M1 A1
ALT	Sub $\left(3t,\frac{3}{t}\right)$ in $2y-8x+45=0\Rightarrow t=-\frac{1}{8}$	M1A1
	Sub $t=-\frac{1}{8}$ in $\left(3t,\frac{3}{t}\right)\Rightarrow\left(-\frac{3}{8},-24\right)$	M1A1
		(4)
		(9 marks)

Notes

(a) M1: Differentiates to obtain $\frac{k}{x^2}$ and substitutes $x = 6$

or uses implicit differentiation $\frac{dy}{dx}=-\frac{y}{x}$ and substitutes x and y

or uses parametric differentiation $\frac{dy}{dx}=-\frac{1}{t^2}$ and substitutes $t=2$

A1: For grad of tangent – accept any equivalent i.e. - 0.25 etc

M1: Uses negative reciprocal of their gradient.

dM1: $y-y_1=m(x-x_1)$ with $\left(6,\frac{3}{2}\right)$ or $y=mx+c$ and sub $\left(6,\frac{3}{2}\right)$ to find c=.

A1: cso: Correct answer with no errors seen in the solution.

(b) M1: Obtains equation in one variable, x or y

A1: Correct value of x or correct value of y

M1: Finds second coordinate using $xy = 9$ or solving second quadratic or equation of the normal

A1: Correct coordinates that can be written as $x=\ldots, y=\ldots$

Question Number	Scheme	Marks
6. (i)	If $n = 1$, $\begin{pmatrix} 1 & 0 \\ -1 & 5 \end{pmatrix}^1 = \begin{pmatrix} 1 & 0 \\ -\frac{1}{4}(5^1-1) & 5^1 \end{pmatrix}$ so **true** for $\boldsymbol{n = 1}$	B1
	Assume result true for $n = k$ $\begin{pmatrix} 1 & 0 \\ -1 & 5 \end{pmatrix}^{k+1} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{4}(5^k-1) & 5^k \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -1 & 5 \end{pmatrix}$ or $\begin{pmatrix} 1 & 0 \\ -1 & 5 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -\frac{1}{4}(5^k-1) & 5^k \end{pmatrix}$	M1
	$\begin{pmatrix} 1 & 0 \\ -1 & 5 \end{pmatrix}^{k+1} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{4}(5^k-1)-5^k & 5\times 5^k \end{pmatrix}$ or $\begin{pmatrix} 1 & 0 \\ -1-5.\frac{1}{4}(5^k-1) & 5\times 5^k \end{pmatrix}$	M1 A1
	$= \begin{pmatrix} 1 & 0 \\ -\frac{1}{4}5^k+\frac{1}{4}-5^k & 5^{k+1} \end{pmatrix}$ or $\begin{pmatrix} 1 & 0 \\ -1-\frac{1}{4}5^{k+1}+\frac{5}{4} & 5^{k+1} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{4}(5^{k+1}-1) & 5^{k+1} \end{pmatrix}$	A1
	True for $\boldsymbol{n = k + 1}$ if **true** for $\boldsymbol{n = k}$, (and **true** for $\boldsymbol{n = 1}$) so **true** by induction for all $\boldsymbol{n \in Z^+}$.	A1cso (6)
(ii)	If $n = 1$, $\sum_{r=1}^{n}(2r-1)^2 = 1$ and $\frac{1}{3}n(4n^2-1) = 1$, so **true** for $\boldsymbol{n = 1}$.	B1
	Assume result true for $n = k$ so $\sum_{r=1}^{k+1}(2r-1)^2 = \frac{1}{3}k(4k^2-1)+(2(k+1)-1)^2$	M1
	$= \sum_{r=1}^{k+1}(2r-1)^2 = \frac{1}{3}(2k+1)\{(2k^2-k)+(3(2k+1))\}$	M1 A1
	$= \frac{1}{3}(2k+1)\{(2k^2+5k+3)\}$ or $\frac{1}{3}(k+1)(4k^2+8k+3)$ or $\frac{1}{3}((2k+3)(2k^2+3k+1)\}$	
	$= \frac{1}{3}(k+1)(2k+1)(2k+3)$ $= \frac{1}{3}(k+1)(4(k+1)^2-1)$	dA1
	True for $\boldsymbol{n = k + 1}$ if **true** for $\boldsymbol{n = k}$, (and **true** for $\boldsymbol{n = 1}$) so **true** by induction for all $\boldsymbol{n \in Z^+}$	A1cso (6) **12 marks**

Notes

(i) B1: Checks $n = 1$ on both sides and states true for $n = 1$ seen anywhere.
M1: Assumes true for $n = k$ and indicates intention to multiply power k by power 1 either way around.
M1: Multiplies matrices. Condone one slip. A1: Correct unsimplified matrix
A1: Intermediate step required cao
A1: cso Makes correct induction statement including at least statements in bold.
Statement **true** for $\boldsymbol{n = 1}$ here could contribute to B1 mark earlier.

(ii) B1: Checks $n = 1$ on both sides and states true for $n = 1$ seen anywhere.
M1: Assumes true for $n = k$ and adds $(k+1)^{th}$ term to sum of k terms. Accept $4(k+1)^2-4(k+1)+1$ or $(2k+1)^2$ for $(k+1)^{th}$ term. M1: Factorises out a linear factor of the three possible - usually $2k+1$
A1: Correct expression with one linear and one quadratic factor.
dA1: Need to see $\frac{1}{3}(k+1)(4(k+1)^2-1)$ somewhere dependent upon previous A1.
Accept assumption plus $(k+1)^{th}$ term and $\frac{1}{3}(k+1)(4(k+1)^2-1)$ both leading to $\frac{1}{3}(4k^3+12k^2+11k+3)$ then award for expressions seen as above.
A1: cso Makes correct complete induction statement including at least statements in bold. Statement true for $n = 1$ here could contribute to B1 mark earlier.

Question Number	Scheme	Marks
7. (i)	$5k(k+1) - -3(3k-1)=0$	M1
	$5k^2+5k+9k-3=0$	A1
	$(5k-1)(k+3)=0$ so $k=$	M1
	$k=\frac{1}{5}$ or -3	A1
		(4)
(ii)(a)	$\mathbf{B}^{-1}=\frac{1}{45}\begin{pmatrix}3 & -5\\ 3 & 10\end{pmatrix}$	M1 A1
		(2)
(b)	$\frac{1}{45}\begin{pmatrix}3 & -5\\ 3 & 10\end{pmatrix}\begin{pmatrix}0 & -20 & 10c\\ 0 & 6 & 6c\end{pmatrix}=$	M1
	$\frac{1}{45}\begin{pmatrix}0 & -90 & 0\\ 0 & 0 & 90c\end{pmatrix}$	
	Vertices at (0, 0) (-2, 0) (0, $2c$)	A1,A1
		(3)
ALT	$\begin{pmatrix}10 & 5\\ -3 & 3\end{pmatrix}\begin{pmatrix}a & d & f\\ b & e & g\end{pmatrix}=\begin{pmatrix}0 & -20 & 10c\\ 0 & 6 & 6c\end{pmatrix}$ and attempt to form simultaneous equations	M1
	$10a+5b=0,-3a+3b=0$	
	$10d+5e=-20,-3d+3e=6$ all correct oe	A1
	$10f+5g=10c,-3f+3g=6c$	
	Vertices at (0, 0) (-2, 0) (0, $2c$)	A1
(c)	Area of T is $\frac{1}{2}\times 2\times 2c=2c$ OR Area of $T'=\frac{1}{2}\begin{vmatrix}0 & -20 & 10c & 0\\ 0 & 6 & 6c & 0\end{vmatrix}=90c$	B1
	Area of $T\times$determinant $=135$ Their area = 135	M1
	So $c=\frac{3}{2}$ So $c=\frac{3}{2}$	A1
		(3)
		(12 marks)

Notes

(i) M1: Puts determinant equal to zero
A1: cao as three or four term quadratic
M1: Solve their quadratic to find k
A1: cao – need both correct answers

(ii) (a) M1 Uses correct method for inverse with fraction $\frac{1}{45}$ or $\frac{1}{\text{their det}}$

A1: All correct oe

(b) M1: Post multiplies their inverse by **2 by 3 matrix or 2 by 2 matrix excluding the origin** or does not use inverse and attempts to form simultaneous equations. Can exclude origin.
A1: (-2,0) and (0,2c).Can be written as column vectors. Accept seen in final two columns of single matrix
A1: (0,0). Can be written as column vectors. Award if seen as first column of single matrix.

(c) B1: Area of T given as $2c$ or area of $T'=90c$ Accept $\pm$
M1: Either method using their area of T and their det or their area of T'
A1: $c=\frac{3}{2}$ cao

Question Number	Scheme	Marks
8(a)	$SP = \sqrt{(3p^2 - a)^2 + 36p^2}$, with $a = 3$	M1, B1
	$SP = \sqrt{9p^4 + 18p^2 + 9}$ $= 3(1+p^2)$ ****given answer****	A1 *
		(3)
ALT	For parabola, perpendicular distance from P to directrix $= SP$	M1
	Directrix $x = -3$	B1
	So $SP = 3 + 3p^2 = 3(1+p^2)$	A1
(b)	$y^2 = 12x \Rightarrow 2y\frac{dy}{dx} = 12$ or $y = \sqrt{12x} \Rightarrow \frac{dy}{dx} = \sqrt{3}x^{-\frac{1}{2}}$ or $\frac{dy}{dx} = \frac{\frac{dy}{dp}}{\frac{dx}{dp}}$ or $\frac{dy}{dx} = \frac{\frac{dy}{dq}}{\frac{dx}{dq}}$	M1
	The tangent at P has gradient $= \frac{1}{p}$ or the tangent at Q has gradient $\frac{1}{q}$	A1
	and equation is $y - 6p = \frac{1}{p}(x - 3p^2)$ or $py = x + 3p^2$ o.e.	A1
	Tangent at Q is $y - 6q = \frac{1}{q}(x - 3q^2)$ or $qy = x + 3q^2$ o.e.	B1
	Eliminate x or y: So $x = 3pq$ or $y = 3(p+q) = 3p + 3q$	M1 A1
	Substitute for second variable so $x = 3pq$ and $y = 3(p+q) = 3p + 3q$	M1 A1
		(8)
(c)	$SR^2 = (3 - 3pq)^2 + (3p + 3q)^2 (= 9 + 9p^2q^2 + 9p^2 + 9q^2)$	M1
	$SP.SQ = 3(1+p^2)\ 3(1+q^2)$ $(= 9 + 9p^2q^2 + 9p^2 + 9q^2)$	M1
	So $SR^2 = SP.SQ$ as required	A1
		(3)
		(14 marks)

Notes

(a) M1: Uses distance between two points or states perpendicular distance from P to directrix required.
B1: States or uses focus at (3,0) or focus at $a = 3$ or directrix as x = -3
A1: cso
(b) M1: Calculus method for finding gradient and substitutes x value at either point
A1: Either correct. Accept unsimplified.
A1: One equation of tangent correct. B1: Both correct
M1: Eliminate x or y. A1: Obtain first variable
M1: Substitute or eliminate again. A1: Both variables correct in simplest form as above.
(c) M1:Find their $SR^2 = (3 - 3pq)^2 + (3p + 3q)^2 (= 9 + 9p^2q^2 + 9p^2 + 9q^2)$
M1: Find their $SP.SQ = 3(1+p^2)\ 3(1+q^2)$ $(= 9 + 9p^2q^2 + 9p^2 + 9q^2)$
A1: Deduce equal after no errors seen. Concluding statement required cso.

Pearson Education Limited. Registered company number 872828
with its registered office at Edinburgh Gate, Harlow, Essex CM20 2JE