GradeMax

FP1 Jan 2014 (IAL) (MA)

Q1a) $f(1) = -0.7015115\ldots$
$f(1.4) = 1.950164\ldots$
} change in sign between $x=1$ and $x=1.4$ ∴ a root lies between $x=1$ and 1.4.

b)

a	f(a)	b	f(b)	$\frac{a+b}{2}$	$f\left(\frac{a+b}{2}\right)$
1	−0.7015..	1.4	1.95016..	1.2	0.588...
1	−0.7015	1.2	0.588...	1.1	−0.06798...

∴ interval is $\boxed{[1.1, 1.2]}$

Q2ai) Area image = Area object × |det A|.

$2x = x \cdot |\det A|$.

$\det A = -4k + 30$

∴ $2x = x(30 - 4k)$

$2 = |30 - 4k|$

$2 = 30 - 4k$ or $2 = -(30 - 4k)$
$4k = 28$ $4k = 32$
$\boxed{k = 7}$ $\boxed{k = 8}$

ii) $BC = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 5 & 1 \end{pmatrix} \begin{pmatrix} 2 & 8 \\ 0 & 2 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} 2+3 & 8-4-6 \\ -4+1 & 10-18 \end{pmatrix}$

② × 3 3 × ② 2 × 2

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$$= \boxed{\begin{pmatrix} 5 & -2 \\ -3 & -8 \end{pmatrix}}$$

Q3) $x = 2t \qquad y = \frac{2}{t}$

$P\ (1, 4) \qquad Q\ (8, \frac{1}{2})$

$$m_{PQ} = \frac{\frac{1}{2} - 4}{8 - 1} = -\frac{1}{2} //$$

$\therefore \quad m_L = 2 //$ $\quad \left(-\frac{1}{2} \times 2 = -1\right)$

↳ L is ⊥ to PQ.

$\Rightarrow \quad y - 0 = 2(x - 0)$

$\Rightarrow \quad \boxed{y = 2x}$

Q4) $f(x) = 2x^{\frac{1}{2}} - 6x^{-2} - 3$

$f'(x) = x^{-\frac{1}{2}} + 12x^{-3}$

$f(3.5) = 0.25186..$

$f'(3.5) = 0.81441...$

$$\Rightarrow x_1 = 3.5 - \frac{0.25186...}{0.81441...} = \boxed{3.191} \text{ 3dp.}$$

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Q5a) $|w| = \sqrt{(\sqrt{3})^2 + (1)^2} = \boxed{2}$

b) $zw = (5+\sqrt{3}i)(\sqrt{3}-i) = 5\sqrt{3} - 5i + 3i + \sqrt{3}$

$= \boxed{6\sqrt{3} - 2i}$

c) $\frac{z}{w} = \frac{5+i\sqrt{3}}{\sqrt{3}-i} = \frac{(5+i\sqrt{3})(\sqrt{3}+i)}{(\sqrt{3}-i)(\sqrt{3}+i)}$

$= \frac{5\sqrt{3} + 5i + 3i - \sqrt{3}}{3+1} = \frac{4\sqrt{3} + 8i}{4}$

$= \boxed{\sqrt{3} + 2i}$

d) $z + \lambda = (5+\lambda) + i\sqrt{3}$.



$\tan\frac{\pi}{3} = \frac{\sqrt{3}}{5+\lambda} = \sqrt{3}$ $(\div\sqrt{3})$ $\frac{1}{5+\lambda} = 1.$

$\Rightarrow \quad 5+\lambda = 1$

$\Rightarrow \quad \boxed{\lambda = -4}$

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Q6a) $\sum_1^n r(r+1)(r-1) = \sum_1^n r(r^2-1) = \sum_1^n r^3 - r$

$= \sum_1^n r^3 - \sum_1^n r$

$= \frac{n^2}{4}(n+1)^2 - \frac{n}{2}(n+1)$

$= \frac{n}{4}(n+1)\left[n(n+1) - 2\right]$

$= \frac{n}{4}(n+1)(n^2+n-2)$

$= \frac{n}{4}(n+1)(n+2)(n-1)$

$a = 2$

b) $10\sum_1^n r^2 = \frac{10}{6}n(n+1)(2n+1)$

$\Rightarrow \frac{10^{\cancel{5}}}{6}(n+1)(2n+1) = \frac{\cancel{4}}{4}(n+1)(n+2)(n-1)$

$\Rightarrow \frac{5}{3}(2n+1) = \frac{1}{4}(n^2+n-2)$

$\times 12$

$\Rightarrow 20(2n+1) = 3n^2 + 3n - 6$

$\Rightarrow 3n^2 - 37n - 26 = 0$

$\Rightarrow (3n+2)(n-13) = 0 \quad n > 1 \quad \therefore \boxed{n = 13}$

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Q7a) $\det P = 3a(2b) - (2ab) = 6ab - 2ab$

$= 4ab$

$\therefore P^{-1} = \boxed{\frac{1}{4ab}\begin{pmatrix} 2b & 2a \\ b & 3a \end{pmatrix}}$ //

b) $M = PQ$.

$P^{-1}M = P^{-1}PQ$

$P^{-1}M = Q$ //

$P^{-1}M = \frac{1}{4ab}\begin{pmatrix} 2b & 2a \\ b & 3a \end{pmatrix}\begin{pmatrix} -6a & 7a \\ 2b & -b \end{pmatrix}$

$= \frac{1}{4ab}\begin{pmatrix} -12ab+4ab & 14ab-2ab \\ -6ab+6ab & -3ab+7ab \end{pmatrix} = \frac{1}{4ab}\begin{pmatrix} -8ab & 12ab \\ 0 & 4ab \end{pmatrix}$

$= \boxed{\begin{pmatrix} -2 & 3 \\ 0 & 1 \end{pmatrix}}$.

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Q8a) $y^2 = 4ax$ [Implicit differentiate]

$2y\frac{dy}{dx} = 4a \quad \therefore \frac{dy}{dx} = \frac{2a}{y} = \frac{2a}{2ap} = \frac{1}{p}$ //

so at normal, $m = -p$ //

$\Rightarrow y - 2ap = -p(x - ap^2)$

$\Rightarrow y = -px + ap^3 + 2ap$

$\Rightarrow y + px = ap^3 + 2ap$

b) $(6a, 0)$: $0 + p(6a) = ap^3 + 2ap$ //

$6ap - 2ap = ap^3$

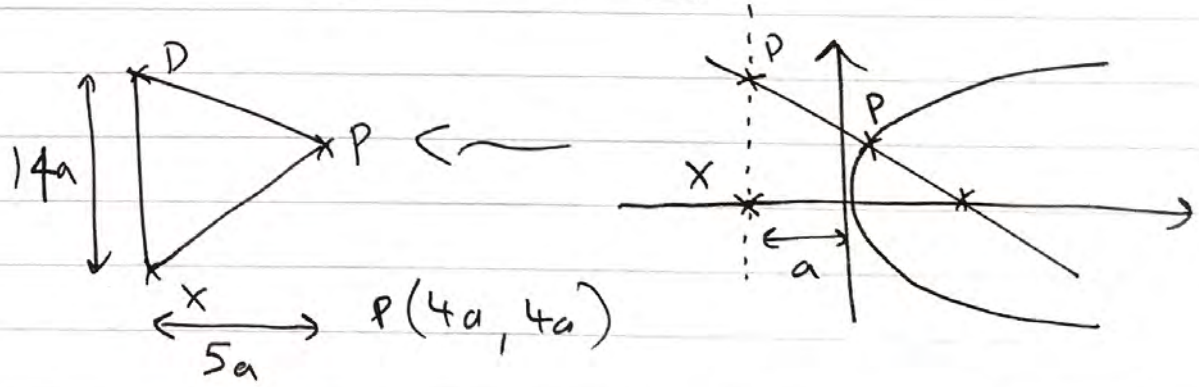
$4 = p^2 \quad \therefore p = 2$ // $(p > 2)$.

at D, $x = -a$: $y - a(2) = a(8) + 2a(2)$

$y = 8a + 4a + 2a = 14a$

so $\boxed{D(-a, 14a)}$

c)



$\therefore$ Area XPD $= \frac{1}{2} \times 5a \times 14a = \boxed{35a^2}$

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Q9) $(3-i)z^* + 2iz = 9-i$

$z^* = x - iy$

$z = x + iy$

$(3-i)(x-iy) + 2i(x+iy) = 9-i$

$3x - 3iy - ix - y + 2xi - 2y = 9-i$

Gather all real / imaginary terms,

$\Rightarrow (3x - y - 2y) + (2x - x - 3y) = 9 - i$

$\Rightarrow (3x - 3y) + (x - 3y) = (9) + (-1)i$

compare real / imaginary terms,

$3x - 3y = 9$ ~ (1)

$x - 3y = -1$ ~ (2)

] solve simultaneously,

(1): $x - y = 3 \quad \therefore \quad y - x = -3$

(2): $x - 3y = -1$

(1) + (2): $y + x - x - 3y = -4$

$\hookrightarrow -2y = -4$

$\therefore \boxed{y = 2}$

So $x = y + 3$

$\boxed{x = 5}$