GradeMax

C12 January 2018 (MA)

Q1 a) $y = \frac{1}{3}x^{\frac{2}{3}} + \frac{1}{2}$

$\frac{dy}{dx} = \frac{2}{3}\left(\frac{1}{3}\right)x^{-\frac{1}{3}} + 0$

$\therefore \boxed{\frac{dy}{dx} = \frac{2}{9}x^{-\frac{1}{3}}}$

b) $\int\left(\frac{1}{3}x^{\frac{2}{3}} + \frac{1}{2}\right)dx = \frac{\frac{1}{3}x^{\frac{5}{3}}}{\frac{5}{3}} + \frac{1}{2}x + c$

$= \boxed{\frac{1}{5}x^{\frac{5}{3}} + \frac{1}{2}x + c}$

Q2 a) $u_2 = 2 - 3(1) = \boxed{-1}$

$u_3 = 2 - 3(-1) = \boxed{5}$

b) $\sum_{r=1}^{4} r - u_r = (1+2+3+4) - (u_1+u_2+u_3+u_4)$

($\sum r$ over $1+2+3+4$; $\sum u_r$ over $u_1+u_2+u_3+u_4$)

$= (10) - (1-1+5+2-3(5))$

$= 10 - -8 = \boxed{18}$

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(Q3a) $(3x^{\frac{1}{2}})^4 = 3^4 \times x^{\frac{1}{2}\times 4} = \boxed{81x^2}$

b) $\dfrac{2y^7 \times (4y)^{-2}}{3y} = \dfrac{2y^7 \times \frac{1}{16y^2}}{3y}$

$= \dfrac{\frac{2y^7}{16y^2}}{3y} = \dfrac{\frac{1}{8}y^5}{3y} = \dfrac{y^5}{24y} = \boxed{\dfrac{y^4}{24}}$

(Q4a) $(p-2)x^2 + (8)x + (p+4) = 0$

no real roots so $b^2 - 4ac < 0$

$a = p-2$, $b = 8$, $c = p+4$

$b^2 - 4ac < 0$

$(8)^2 - 4(p-2)(p+4) < 0$

$64 - 4(p^2 + 2p - 8) < 0$

$64 - 4p^2 - 8p + 32 < 0$

$-4p^2 - 8p + 96 < 0$

$\div 4$: $-p^2 - 2p + 24 < 0$

$\times -1$: $p^2 + 2p - 24 > 0$

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b) $p^2 + 2p - 24 = 0$

finding critical values ... By Quadratic Formula

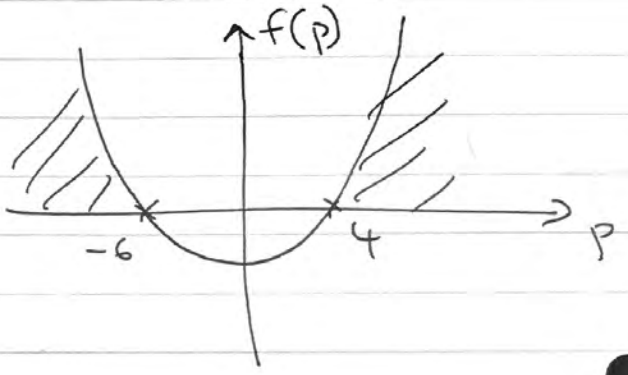
$a = 1$
$b = 2$
$c = -24$

$$p = \frac{-2 \pm \sqrt{4 - 4(-24)}}{2}$$

$$p = \frac{-2 \pm \sqrt{100}}{2}$$

So $p = 4$, $p = -6$

we want where $y > 0$.



So $\boxed{\begin{array}{l} p > 4 \\ p < -6 \end{array}}$

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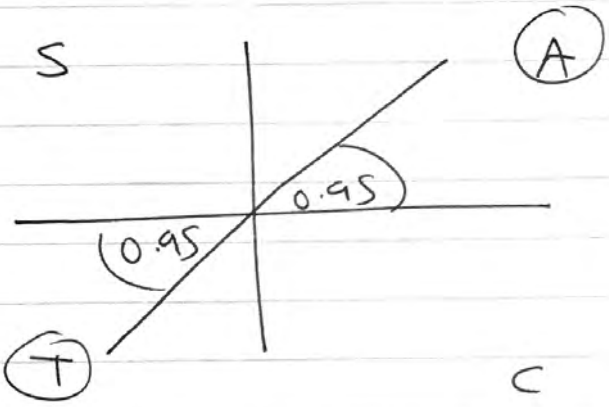
(Q5 i) $5\sin 3\theta - 7\cos 3\theta = 0$

$\div \cos 3\theta$: $5\tan 3\theta = 7$

$$\tan 3\theta = \frac{7}{5}$$

$$3\theta = \tan^{-1}\left(\frac{7}{5}\right) = 0.9505\ldots$$

range : $0 < 3\theta < \frac{3\pi}{2}$



$3\theta = 0.9505, \pi + 0.9505$

$3\theta = 0.9505, 4.092$

so $\theta = \boxed{0.317, 1.36}$

ii) $9\cos^2 x + 5\cos x = 3 - 3\cos^2 x$

$12\cos^2 x + 5\cos x - 3 = 0$

$(3\cos x - 1)(4\cos x + 3) = 0$

$3\cos x - 1 = 0$

$\cos x = \frac{1}{3}$ //

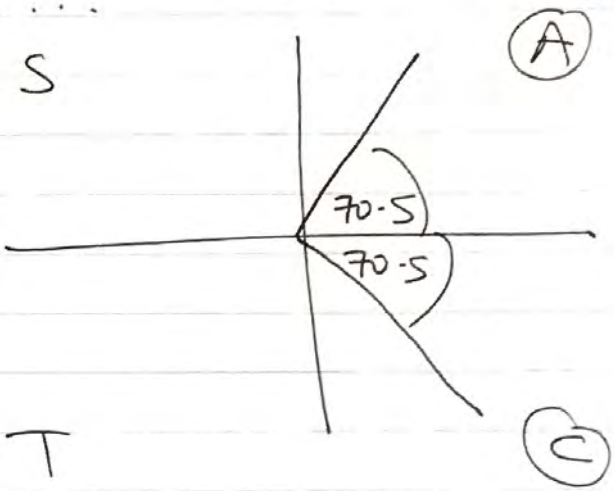
$4\cos x + 3 = 0$

$\cos x = -\frac{3}{4}$ //

$x = \cos^{-1}\left(\frac{1}{3}\right) = 70.53°\ldots$

$x = 70.5°, 360 - 70.5°$

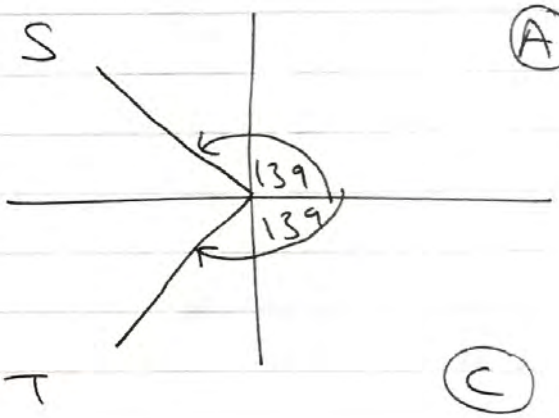
$\boxed{x = 70.5°, 289.5°}$



$x = \cos^{-1}\left(-\frac{3}{4}\right) = 138.6°$

$x = 138.6°, 360 - 138.6°$

$\boxed{x = 138.6°, 221.4°}$



Q6a) $f(x)$ div by $(x+1) \rightarrow$ no remainder

So $f(-1) = 0$.

$f(-1) = -a - 8 - b + 6 = 0$

$a + b = -2$ //

$f(x)$ div by $(x-2) \rightarrow$ remainder $= -12$

So $f(2) = -12$

↳ $f(2) = 8a - 32 + 2b + 6 = -12$ //

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$\therefore 8a + 2b = 14$

$\div 2: \quad 4a + b = 7$

$-\ (a + b = -2)$

$3a + 0 = 9$

$\therefore 3a = 9 \quad$ so $\boxed{a = 3}$

$b = -2 - a = -2 - 3 = \boxed{-5}$

b)

$$
\begin{array}{r|l}
 & 3x^2 - 11x + 6 \\
\hline
x + 1 & 3x^3 - 8x^2 - 5x + 6 \\
 & 3x^3 + 3x^2 \\
\hline
 & 0 - 11x^2 - 5x \\
 & -11x^2 - 11x \\
\hline
 & 0 + 6x + 6 \\
 & 6x + 6 \\
\hline
 & 0 \quad 0
\end{array}
$$

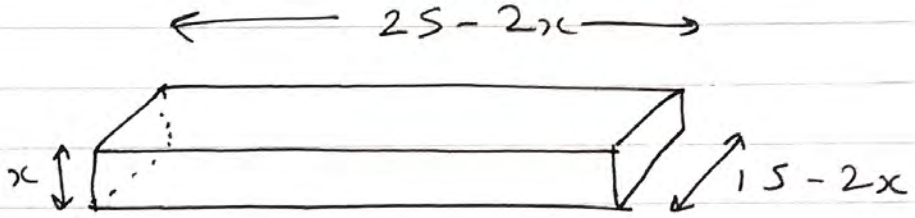
$\therefore f(x) = (x + 1)(3x^2 - 11x + 6)$

but $(3x^2 - 11x + 6) = (3x - 2)(x - 3)$

$\therefore f(x) = \boxed{(x + 1)(3x - 2)(x - 3)}$

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(Q7a)



$$\text{Volume} = (25-2x)\times(15-2x)\times(x)$$

$$V = (375 - 50x - 30x + 4x^2)\times x$$

$$V = (375 - 80x + 4x^2)\times x$$

$$V = 4x^3 - 80x^2 + 375x$$

b) $$\frac{dV}{dx} = 12x^2 - 160x + 375$$

at max value, $\frac{dV}{dx} = 0$.

$$\Rightarrow 12x^2 - 160x + 375 = 0$$

By Quadratic formula...

$a = 12$
$b = -160$
$c = 375$

$x = 3.03,\ 10.3$

but $x \neq 10.3$ since side length is $15 - 2x$

so $\boxed{x = 3.03}$

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c) $\frac{d^2V}{dx^2} = 24x - 160$

$x = 3.03 \rightarrow \left(\frac{d^2V}{dx^2}\right)_{x=3.03} = 24(3.03) - 160$

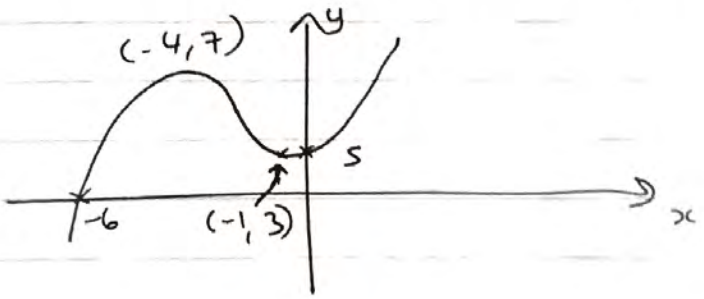
$\approx -87.28 < 0$

hence value is a maximum.

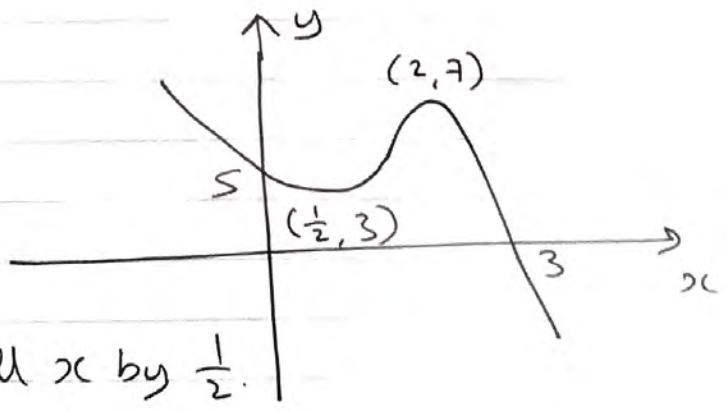
d) $V = 4(3.03)^3 - 80(3.03)^2 + 375(3.03)$

$\approx \boxed{513}\ cm^3$

(Q8a) $f(-x)$ → multiply all x by -1



b) $f(2x)$



multiply all x by $\frac{1}{2}$.

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(Q9a) a ar ar^2

20 $20(0.9)$ $20(0.9)^2$

5^{th} term $= ar^4 = 20 \times (0.9)^4$

$= \boxed{13.122}$

b) $S_8 = \dfrac{20(1-0.9^8)}{1-0.9} = \boxed{113.9}$

c) $S_\infty = \dfrac{a}{1-r} = \dfrac{20}{1-0.9} = 200$

$S_\infty - S_N = 200 - \dfrac{20(1-0.9^N)}{0.1}$

$S_\infty - S_N = 200 - 200(1-0.9^N)$

$= 200 - 200 + 200(0.9^N)$

$= 200(0.9^N)$ //

hence $200(0.9^N) < 0.04$

$\div 200$: $0.9^N < \dfrac{0.04}{200}$

$\therefore \quad 0.9^N < 0.0002$ //

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d) $\log(0.9^N) < \log(0.0002)$

$N \log(0.9) < \log(0.0002)$

$$N > \frac{\log 0.0002}{\log 0.9}$$

÷ by a -ve value flips the signs

$N > 80.84\ldots$

So $\boxed{N_{min} = 81}$

(Q10.) $3\log_8(2) + \log_8(7-x) = 2 + \log_8(x)$

$\log_8(2^3) + \log_8(7-x) - \log_8(x) = 2$

$\log_8(8)^1 + \log_8\left(\frac{7-x}{x}\right) = 2$

$1 + \log_8\left(\frac{7-x}{x}\right) = 2$

$\therefore \log_8\left(\frac{7-x}{x}\right) = 1$

$\therefore 8^1 = \frac{7-x}{x}$

So $8x = 7 - x$

$9x = 7$ $\therefore \boxed{x = \frac{7}{9}}$

ii) $3^{2y} + 3^{y+1} = 10$

$3^{2y} + (3^y)(3) - 10 = 0$

let $3^y = u$,

then $u^2 + 3u - 10 = 0$

$(u+5)(u-2) = 0$

$u = -5$

$3^y = -5$

$y \log 3 = \log(-5)$

↙

no valid solutions....

($a^x > 0$ for all x)

$u = 2$

$3^y = 2$

$\log(3^y) = \log(2)$

$y \log 3 = \log 2$

$y = \boxed{\frac{\log 2}{\log 3}}$ //

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Q11ai) (ii)

$$x^2 - 8x + y^2 - 10y + 16 = 0$$

$$(x-4)^2 - 16 + (y-5)^2 - 25 + 16 = 0$$

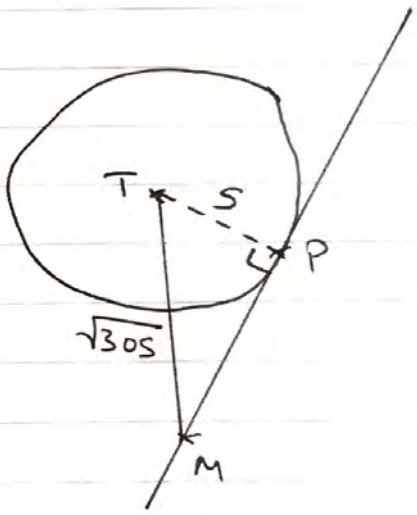
$$\therefore \boxed{(x-4)^2 + (y-5)^2 = 25}$$

so $\boxed{T(4,5)}$

$\boxed{r = 5}$

b) $MT = \sqrt{(4-20)^2 + (5-12)^2} = \boxed{\sqrt{305}}$

c) $PM = \sqrt{(305) - (5)^2} = 2\sqrt{70}$ //



$\therefore \Delta PMT$ area $= \frac{1}{2}(2\sqrt{70})(5)$

$= \boxed{5\sqrt{70}}$

Q12a) $x = -1$: $-1 + 3y - 11 = 0$

$3y = 12$ $\therefore y = \boxed{4 = p}$

$y = 2$: $x + 6 - 11 = 0$

$x = 5$ $\therefore \boxed{q = 5}$

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b) $(-1, 4)$ $(5, 2)$

$$AB = \sqrt{(5--1)^2 + (2-4)^2} = \boxed{2\sqrt{10}}$$

c) midpoint : $\left(\frac{-1+5}{2}, \frac{4+2}{2}\right)$

$$\Rightarrow \left(2, 3\right) //$$

$$m_{AB} = \frac{2-4}{5--1} = \frac{-2}{6} = -\frac{1}{3} //$$

so $m_{L_2} = 3 //$ $\left(3 \times -\frac{1}{3} = -1\right)$

since L_2 is perp to L_1.

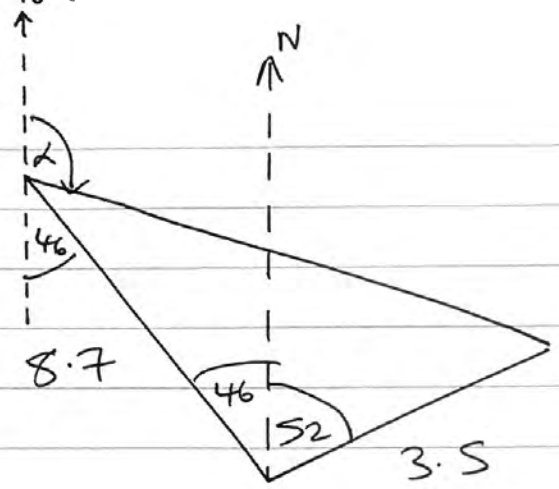
$$\Rightarrow y - 3 = 3(x - 2)$$

$$\Rightarrow y = 3x - 6 + 3$$

$$\Rightarrow \boxed{y = 3x - 3}$$

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Q13a)



$360 - 314 = 46$

$\angle APB = 46 + 52 = \boxed{98^\circ}$

b) cosine rule

$AB^2 = (8.7)^2 + (3.5)^2 - 2(8.7)(3.5)\cos 98$

$\approx 96.42\ldots$

so $AB = \boxed{9.8\text{ km}}$

c) sine rule to find $\angle PAB$.

$$\frac{9.8}{\sin 98} = \frac{3.5}{\sin \angle PAB}$$

$$\therefore \sin \angle PAB = \frac{3.5}{9.8}\sin 98$$

$$\therefore \angle PAB = \sin^{-1}\left(\frac{3.5}{9.8}\sin 98\right) \approx 20.66^\circ$$

Bearing $= \alpha = 180 - 46 - 20.66$

$= \boxed{113^\circ}$

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(Q14a) $8 - x = 14 + 3x - 2x^2$

$2x^2 - 4x - 6 = 0$

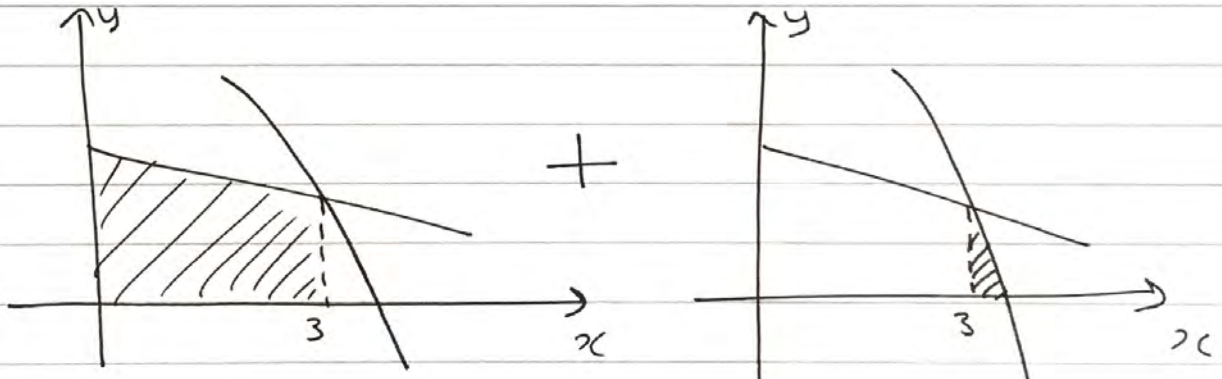
$x^2 - 2x - 3 = 0$

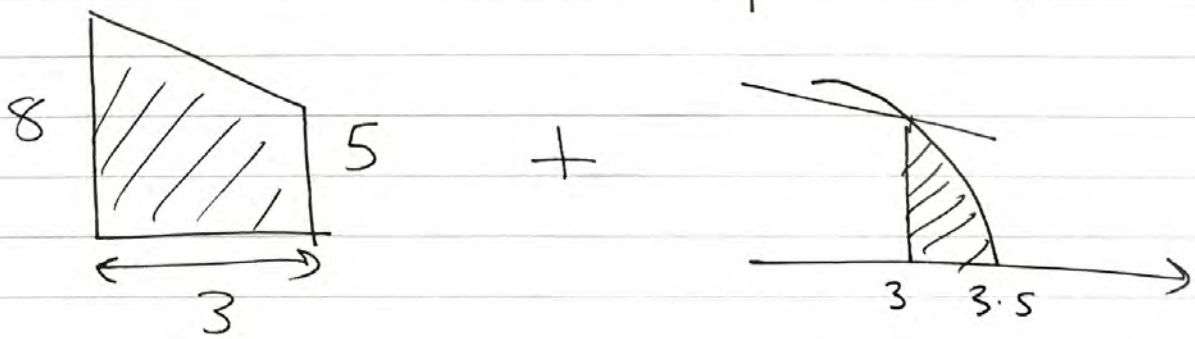
$(x - 3)(x + 1) = 0$

$x = 3$ ↓ $y = 8 - 3 = 5$ so $\boxed{B(3, 5)}$

$x = -1$ ↓ $y = 8 - -1 = 9$ so $\boxed{A(-1, 9)}$

b)

$R =$ 



$14 + 3x - 2x^2 = 0 \longrightarrow x = 3.5$, $x = \frac{2}{7}$

Trapezium Area ① $= \frac{(8+5)(3)}{2} = \frac{39}{2}$ //

Area ② $= \int_3^{3.5} \left[14 + 3x - 2x^2\right] dx$

$= \left[14x + \frac{3}{2}x^2 - \frac{2x^3}{3}\right]_3^{3.5}$

$= \left[\frac{931}{24}\right] - \left[\frac{75}{2}\right] = \frac{31}{24}$ //

$\therefore R = \frac{39}{2} + \frac{31}{24} = \boxed{\frac{499}{24}}$

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Q15a) $(1+kx)^n \equiv 1 + nkx + \frac{n(n-1)k^2x^2}{2}$

$= 1 + 36x + 126kx^2$

so... $36 = nk$ and $\frac{n(n-1)k^2}{2} = 126k$

$\therefore \frac{n(n-1)k}{2} = 126$

hence (×2) $nk(n-1) = 252$

b) $nk(n-1) = 252$

but $nk = 36$

$\therefore 36(n-1) = 252$

$n - 1 = \frac{252}{36}$

$n - 1 = 7 \quad \therefore \boxed{n = 8}$

and $k = \frac{36}{n} = \frac{36}{8} = \boxed{\frac{9}{2}}$

c) x^3 term $= \frac{n(n-1)(n-2)(k^3x^3)}{6}$

so coefficient $= \frac{8(8-1)(8-2)}{6} \times \left(\frac{9}{2}\right)^3 = \boxed{5103}$