GradeMax

C12 October 2016 (MA)

Q1) $\int f(x)\,dx = \int (3x^2 + x - 4x^{-\frac{1}{2}} + 6x^{-3})\,dx$

$= x^3 + \frac{x^2}{2} - \frac{4x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{6x^{-2}}{-2} + c$

$= \boxed{x^3 + \frac{x^2}{2} - 8x^{\frac{1}{2}} - 3x^{-2} + c}$

Q2a) $7^{2x} = 14$

$\log(7^{2x}) = \log(14)$

$2x \log 7 = \log 14$

$2x = \frac{\log 14}{\log 7} \quad \therefore x = \frac{1}{2} \times \frac{\log 14}{\log 7} \approx \boxed{0.678}$

b) $\log_5(3x+1) = -2$

$5^{-2} = 3x + 1$

$\frac{5^{-2} - 1}{3} = x = \frac{\frac{1}{25} - 1}{3} = \boxed{-\frac{8}{25}}$

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(Q3 i) $\sqrt{45} - \frac{20}{\sqrt{5}} + \sqrt{6} \cdot \sqrt{30} = \sqrt{45} - \frac{20\sqrt{5}}{5} + \sqrt{6 \times 30}$

$= \sqrt{9} \times \sqrt{5} - (\sqrt{5} \times 4) + \sqrt{180}$

$= 3\sqrt{5} - 4\sqrt{5} + \sqrt{5} \times \sqrt{36}$

$= 3\sqrt{5} - 4\sqrt{5} + 6\sqrt{5} = (6 + 3 - 4)\sqrt{5}$

$= \boxed{5\sqrt{5}}$

ii) $\frac{17\sqrt{2}}{\sqrt{2} + 6} \times \frac{(\sqrt{2} - 6)}{(\sqrt{2} - 6)}$

$= \frac{17\sqrt{2}(\sqrt{2} - 6)}{(\sqrt{2} + 6)(\sqrt{2} - 6)} = \frac{17(2) - 6(17)\sqrt{2}}{2 - 6\sqrt{2} + 6\sqrt{2} - 36}$

$= \frac{34 - 102\sqrt{2}}{-34} = \boxed{-1 + 3\sqrt{2}}$

(Q4 a i) $2x + 1 = 0$

$2x = -1$

$x = -\frac{1}{2}$ // so $f(-\frac{1}{2}) = 6(-\frac{1}{2})^3 - 7(-\frac{1}{2})^2 - 43(-\frac{1}{2}) + 30$

$= \boxed{49}$

ii) $x - 3 = 0$

$x = 3$ // so $f(3) = 6(3)^3 - 7(3)^2 - 43(3) + 30$

$= -30 + 30 = \boxed{0}$

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b) $f(3) = 0$ so $(x-3)$ is a factor.

$$
\begin{array}{r|l}
x-3 & 6x^2+11x-10 \\
\hline
 & 6x^3-7x^2-43x+30 \\
 & 6x^3-18x^2 \\
\hline
 & 0+11x^2-43x \\
 & 11x^2-33x \\
\hline
 & 0-10x+30 \\
 & -10x+30 \\
\hline
 & 0 \quad 0
\end{array}
$$

$\therefore f(x) = (x-3)(6x^2+11x-10)$

but $6x^2+11x-10 = (2x+5)(3x-2)$

so $f(x) = \boxed{(x-3)(2x+5)(3x-2)}$

Q5a) $\left(3-\frac{ax}{2}\right)^5 \simeq (3)^5 + \binom{5}{1}(3)^4\left(-\frac{ax}{2}\right)^1$

$+ \binom{5}{2}(3)^3\left(-\frac{ax}{2}\right)^2$

$+ \binom{5}{3}(3)^2\left(-\frac{ax}{2}\right)^3$

$\simeq \boxed{243 - \frac{405ax}{2} + \frac{135a^2x^2}{2} - \frac{45}{4}a^3x^3}$

b) coeff. of x is $-\frac{405}{2}a$

coeff. of x^3 is $-\frac{45}{4}a^3$

they are equal so $-\frac{405}{2}a = -\frac{45}{4}a^3$

$\Rightarrow -\frac{405}{2} = -\frac{45}{4}a^2$

$\Rightarrow a^2 = \frac{\frac{405}{2}}{\frac{45}{4}} = 18$ //

$\therefore a = \sqrt{18} = \boxed{3\sqrt{2}}$ $(a > 0)$.

Q6 a) $u_2 = \frac{2}{3}(36) = \boxed{24}$

$u_3 = \frac{2}{3}(24) = \boxed{16}$

$u_4 = \frac{2}{3}(16) = \boxed{\frac{32}{3}}$

} Geometric series!

$a = 36$

$r = \frac{2}{3}$

b) $\boxed{\frac{2}{3}}$

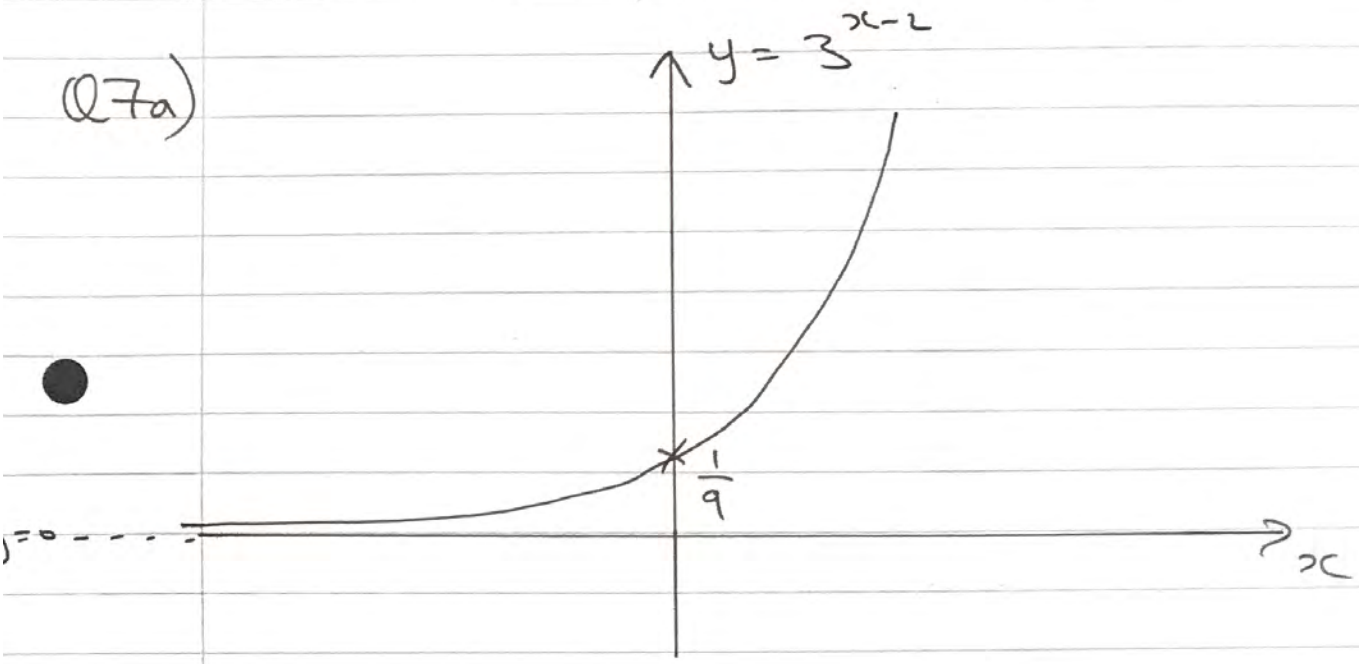
c) $u_{11} = ar^{11-1} = 36 \times \left(\frac{2}{3}\right)^{10} = \boxed{0.6243}$

d) $S_6 = \frac{a(1-r^6)}{1-r} = \frac{36\left(1-\left(\frac{2}{3}\right)^6\right)}{1-\frac{2}{3}} = \boxed{\frac{2660}{27}}$

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e) $S_\infty = \frac{a}{1-r} = \frac{36}{1-\frac{2}{3}} = \boxed{108}$

(Q7a)



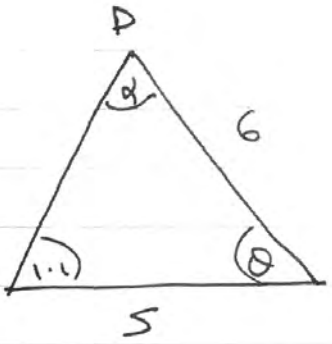
b) $h = \frac{b-a}{n} = \frac{3-0.5}{5} = \frac{2.5}{5} = \frac{1}{2}$

$\therefore$ Area $\simeq \frac{1}{2} \times \frac{1}{2} [0.192 + 3 + 2(0.333 + 0.577 + 1 + 1.732)]$

$\simeq \boxed{2.62}$

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(Q8a) sine rule



$$\frac{\sin\alpha}{5} = \frac{\sin 1.1}{6}$$

$$\sin\alpha = \frac{5}{6}\sin 1.1$$

$$\alpha = \sin^{-1}\left(\frac{5}{6}\sin 1.1\right)$$

so θ = angle required $= \pi - 1.1 - \sin^{-1}\left(\frac{5}{6}\sin 1.1\right)$

$$= \boxed{1.20}$$

b) $\angle DBC = \pi - 1.20 = 1.94$

so Area BCD $= \frac{1}{2}(6)^2(1.94) = 34.9$

and Area ABD $= \frac{1}{2}(6)(5)\sin(1.20) = 14.0$

$34.9 + 14.0 = \boxed{48.9}\ cm^2$

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(Q9a) Sum of first 10 terms $= \frac{10}{2}(2a + 9d) = 395$

$\Rightarrow 5(2a + 9d) = 395$

$\Rightarrow 10a + 45d = 395 \sim (1)$

b) $S_{18} = \frac{18}{2}(2a + 17d) = 927$

$18a + 153d = 927 \sim (2)$

c) (2) : $a = \frac{927 - 153d}{18}$

↳ (1) : $10\left(\frac{927 - 153d}{18}\right) + 45d = 395$

$515 - 85d + 45d = 395$

$-40d = -120$

$\therefore d = \frac{120}{40} = \boxed{3}$

so $a = \frac{927 - 153(3)}{18} = \boxed{26}$

d) n^{th} term $= a + (n-1)d$

20^{th} term $= 26 + 19(3) = \boxed{83}$

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Q10a) $\frac{8\sin x}{\cos x} = -3\cos x$

$\times \cos x$: $8\sin x = -3\cos^2 x$

$8\sin x = -3(1-\sin^2 x)$

$8\sin x = -3 + 3\sin^2 x$

$\therefore \quad 3\sin^2 x - 8\sin x - 3 = 0$

b) Same eqn as in (a) but 2θ instead of x.

$\Rightarrow 3\sin^2(2\theta) - 8\sin(2\theta) - 3 = 0$

By Quadratic formula...

$a = 3$
$b = -8$
$c = -3$

$[\sin 2\theta = 3]$ or $[\sin 2\theta = -\frac{1}{3}]$

NO SOLUTIONS

$\sin 2\theta \leq 1$

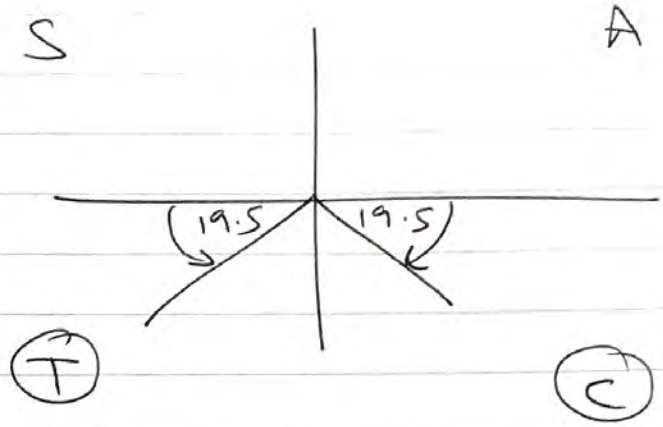
$\sin 2\theta = -\frac{1}{3}$

$\sin^{-1}(-\frac{1}{3}) = 2\theta = -19.47°$ //

range : $0 \leq 2\theta < 720°$

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$2\theta = 180 + 19.5,$
$360 - 19.5,$
$540 + 19.5,$
$720 - 19.5,$



so $2\theta = 199.47°, 340.53°, 559.47°, 700.53°$

$$\theta = 99.7°, 170.3°, 279.7°, 350.3°$$

Q11a) $5x^2 + 6 = k(13x^2 - 12x)$

$5x^2 - 13kx^2 + 6 = -12kx$

$(5 - 13k)x^2 + (12k)x + 6 = 0$

But this eqn has two distinct real roots,

so $b^2 - 4ac > 0$

$a = 5 - 13k$
$b = 12k$
$c = 6$

$b^2 - 4ac = (12k)^2 - 4(5 - 13k)(6)$
$= 144k^2 - 24(5) + 24(13k)$

$\therefore b^2 - 4ac = 144k^2 + 312k - 120$

$b^2 - 4ac > 0$ hence $144k^2 + 312k - 120 > 0$

$\div 24: \quad 6k^2 + 13k - 5 > 0$

b) $6u^2 + 13u - 5 > 0$

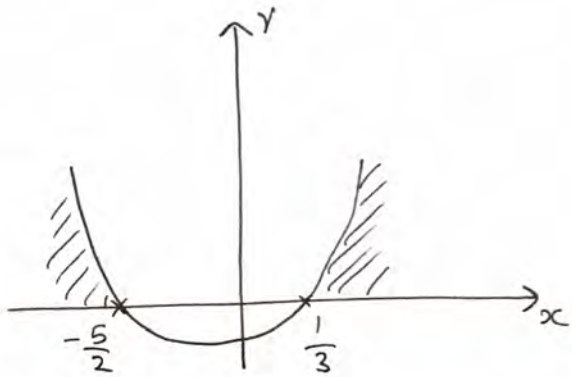
finding critical values... (Solve $6u^2 + 13u - 5 = 0$)

$a = 6$
$b = 13$
$c = -5$

By Quadratic Formula,

$u = \frac{1}{3}$, $u = \frac{-5}{2}$

So $y = 6u^2 + 13u - 5$



$6u^2 + 13u - 5 > 0$ represented by the regions where $y > 0$.

$\therefore$

$u > \frac{1}{3}$

$u < \frac{-5}{2}$

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(Q12a) $y=0$: $\quad 0 = \dfrac{x^3 - 9x^2 - 81x}{27}$

$$\Rightarrow \quad x^3 - 9x^2 - 81x = 0$$

$$\Rightarrow \quad x(x^2 - 9x - 81) = 0$$

$x \neq 0$ so $\quad x^2 - 9x - 81 = 0$
(at A & B)

By Quadratic formula... $\quad x = \dfrac{9 \pm 9\sqrt{5}}{2}$

$(a=1,\ b=-9,\ c=-81)$

So $\quad A\left(\dfrac{9-9\sqrt{5}}{2}, 0\right)$

$B\left(\dfrac{9+9\sqrt{5}}{2}, 0\right)$

b) $y = \dfrac{1}{27}(x^3 - 9x^2 - 81x)$

at C / D, $\dfrac{dy}{dx} = 0$ as they are turning points.

$$\frac{dy}{dx} = \frac{1}{27}(3x^2 - 18x - 81) = 0$$

$$3x^2 - 18x - 81 = 0$$

$$(3x - 27)(x + 3) = 0$$

$(0 = 3x - 27) \rightarrow x = 9$ $\quad (x + 3 = 0) \rightarrow x = -3$

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so C has x coordinate -3.

$$f(-3) = \frac{(-3)^3 - 9(-3)^2 - 81(-3)}{27}$$

$$= 5 //$$

$$\therefore \boxed{C(-3, 5)}$$

and D has x coordinate 9

$$f(9) = \frac{(9)^3 - 9(9)^2 - 81(9)}{27}$$

$$= -27 //$$

$$\therefore \boxed{D(9, -27)}$$

c) D is transformed to the y-axis.
ie 9 units to the left.

so $\boxed{a = 9}$

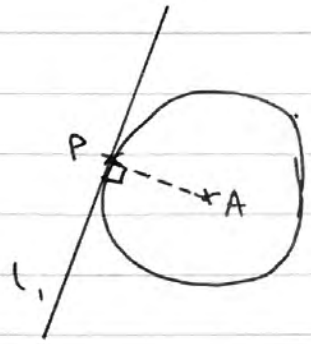
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(Q13a) distance $AP = \sqrt{(1-8)^2 + (-3+2)^2} = \sqrt{50}$ //

$\therefore r = \sqrt{50}$

so $\boxed{(x-1)^2 + (y+3)^2 = 50}$

b) l_1 is perpendicular to AP.



$m_{AP} = \frac{-2--3}{8-1} = \frac{1}{7}$ //

so $m_{l_1} = -7$ // $\left(-7 \times \frac{1}{7} = -1\right)$

$\therefore y--2 = -7(x-8)$

$y = -7x + 56 - 2$

$\boxed{y = -7x + 54}$

c) sub $y = x+6$ into eqn for C.

$(x-1)^2 + (x+6+3)^2 = 50$

$(x-1)^2 + (x+9)^2 = 50$

$x^2 - 2x + 1 + x^2 + 18x + 81 = 50$

$2x^2 + 16x + 82 - 50 = 0$

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$2x^2 + 16x + 32 = 0$

$x^2 + 8x + 16 = 0$

$(x+4)^2 = 0$

$x = -4$ //

at $x = -4$, $y = (-4) + 6 = 2$ //

so $\boxed{Q(-4, 2)}$

Q14a) $y = -x^2 + 6x - 8$

$\frac{dy}{dx} = -2x + 6$

at P, $m = -2(5) + 6 = 6 - 10 = -4$ //

so at nomcl to P, $m = \frac{1}{4}$ // $\left(\frac{1}{4} \times -4 = -1\right)$

$y - -3 = \frac{1}{4}(x - 5)$

$y + 3 = \frac{1}{4}x - \frac{5}{4}$

$y = \frac{1}{4}x - \frac{17}{4}$

$\times 4$: $4y = x - 17$

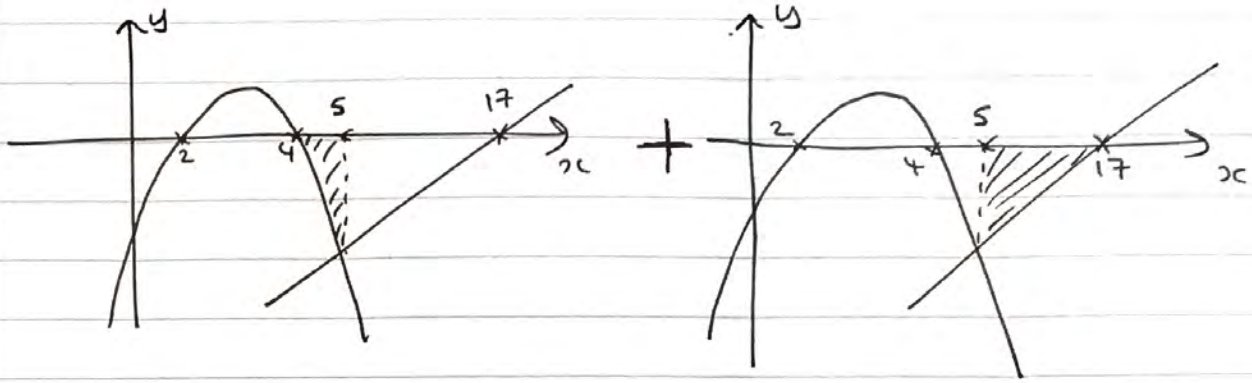
$\boxed{x - 4y - 17 = 0}$

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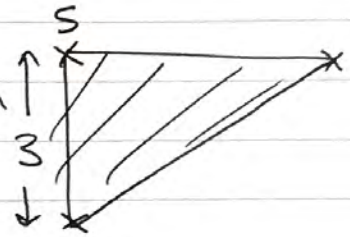
at $y = 0$, $x = 4$, $x = 2$

b)

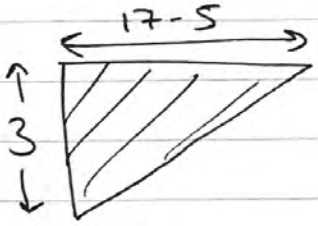
$R =$



$$R = \int_4^5 (-x^2 + 6x - 8)\,dx + \text{Area}$$



$$= \left[\frac{-x^3}{3} + 3x^2 - 8x\right]_4^5 +$$



$$= \left[-\frac{20}{3}\right] - \left[-\frac{16}{3}\right] + \frac{1}{2}(3)(12)$$

$$= \left|-\frac{4}{3}\right| + \frac{36}{2} = 18 + \frac{4}{3}$$

$$= \boxed{\frac{58}{3}}$$

(Q15a) Surface Area $= \frac{\pi r^2}{2} + \frac{\pi r^2}{2} + \frac{1}{2}(2\pi rh) + h(2r)$

$= \pi r^2 + \pi rh + 2hr$ //

$\therefore 200 = \pi r^2 + \pi rh + 2hr$

the Volume equation we need to prove does not include h, so make h the subject here:

$200 = \pi r^2 + h(\pi r + 2r)$

$h = \frac{200 - \pi r^2}{r(\pi + 2)}$ //

and $V = \frac{\pi r^2 h}{2} = \frac{\pi r^2}{2}\left(\frac{200 - \pi r^2}{r(\pi + 2)}\right)$

$\Rightarrow V = \frac{\pi r^{\cancel{2}}(200 - \pi r^2)}{2\cancel{r}(\pi + 2)} = \frac{\pi r(200 - \pi r^2)}{4 + 2\pi}$

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just a constant.

b) $V = \left(\frac{\pi}{4+2\pi}\right)\left[200r - \pi r^3\right]$

$\frac{dV}{dr} = \frac{\pi}{4+2\pi}\left[200 - 3\pi r^2\right] = 0$

(at max value, $\frac{dV}{dr} = 0$)

$\Rightarrow 200 - 3\pi r^2 = 0$

$\Rightarrow 200 = 3\pi r^2$

$\Rightarrow r^2 = \frac{200}{3\pi}$

$\Rightarrow r = \sqrt{\frac{200}{3\pi}} \quad (= 4.6)$

$\text{so } V_{max} = \frac{\pi}{4+2\pi}\left[200\sqrt{\frac{200}{3\pi}} - \pi\left(\sqrt{\frac{200}{3\pi}}\right)^3\right]$

$= \boxed{188} \text{ cm}^3$

c) $\frac{d^2V}{dr^2} = \frac{\pi}{4+2\pi}\left[-6\pi r\right] < 0$

for all r

hence value of V found is a maximum.