Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
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Centre Number | Candidate Number

# Pearson Edexcel International Advanced Level

## Thursday 30 May 2024

Morning (Time: 1 hour 30 minutes)

Paper reference **WMA13/01**

# Mathematics

## International Advanced Level
## Pure Mathematics P3

**You must have:**
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

*Turn over* ▶

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Pearson

**1.**

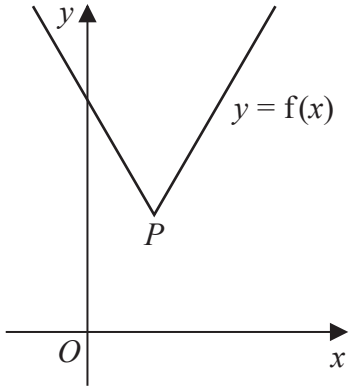



**Figure 1**

Figure 1 shows a sketch of the graph with equation $y = f(x)$ where

$$f(x) = 2|x - 5| + 10$$

The point $P$, shown in Figure 1, is the vertex of the graph.

(a) State the coordinates of $P$

**(2)**

(b) Use algebra to solve

$$2|x - 5| + 10 > 6x$$

(*Solutions relying on calculator technology are not acceptable.*)

**(2)**

(c) Find the point to which $P$ is mapped, when the graph with equation $y = f(x)$ is transformed to the graph with equation $y = 3f(x - 2)$

**(2)**

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**Question 1 continued**

**(Total for Question 1 is 6 marks)**

**2.** $$g(x) = \frac{2x^2 - 5x + 8}{x - 2}$$

(a) Write $g(x)$ in the form

$$Ax + B + \frac{C}{x - 2}$$

where $A$, $B$ and $C$ are integers to be found.

**(3)**

(b) Hence use algebraic integration to show that

$$\int_4^8 g(x)\,dx = \alpha + \beta \ln 3$$

where $\alpha$ and $\beta$ are integers to be found.

**(4)**

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**Question 2 continued**

**Question 2 continued**

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**Question 2 continued**

**(Total for Question 2 is 7 marks)**

**3.** (i) The variables $x$ and $y$ are connected by the equation

$$y = \frac{10^6}{x^3} \qquad x > 0$$

Sketch the graph of $\log_{10} y$ against $\log_{10} x$

Show on your sketch the coordinates of the points of intersection of the graph with the axes.

**(3)**

(ii)

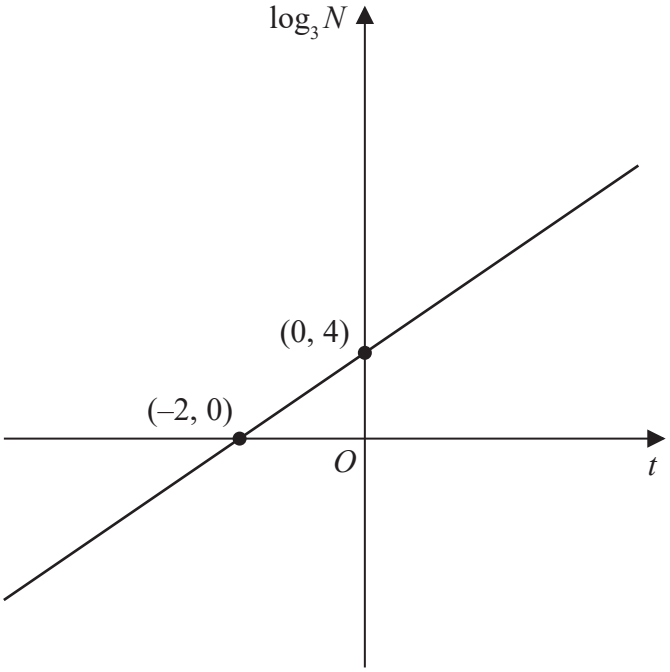



**Figure 2**

Figure 2 shows the linear relationship between $\log_3 N$ and $t$.

Show that $N = ab^t$ where $a$ and $b$ are constants to be found.

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**Question 3 continued**

**(Total for Question 3 is 6 marks)**

**4.** $f(x) = 8\sin x\cos x + 4\cos^2 x - 3$

(a) Write $f(x)$ in the form

$$a\sin 2x + b\cos 2x + c$$

where $a$, $b$ and $c$ are integers to be found.

**(3)**

(b) Use the answer to part (a) to write $f(x)$ in the form

$$R\sin(2x + \alpha) + c$$

where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of $R$ and give the value of $\alpha$ in radians to 3 significant figures.

**(3)**

(c) Hence, or otherwise,

(i) state the maximum value of $f(x)$

(ii) find the **second** smallest positive value of $x$ at which a maximum value of $f(x)$ occurs. Give your answer to 3 significant figures.

**(3)**

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**Question 4 continued**

**Question 4 continued**

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**Question 4 continued**

**(Total for Question 4 is 9 marks)**

**5.** The functions f and g are defined by

$$f(x) = 2 + 5\ln x \qquad x > 0$$

$$g(x) = \frac{6x - 2}{2x + 1} \qquad x > \frac{1}{3}$$

(a) Find $f^{-1}(22)$

**(2)**

(b) Use differentiation to prove that g is an increasing function.

**(3)**

(c) Find $g^{-1}$

**(3)**

(d) Find the range of fg

**(2)**

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**Question 5 continued**

**Question 5 continued**

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**Question 5 continued**

**(Total for Question 5 is 10 marks)**

**6.**

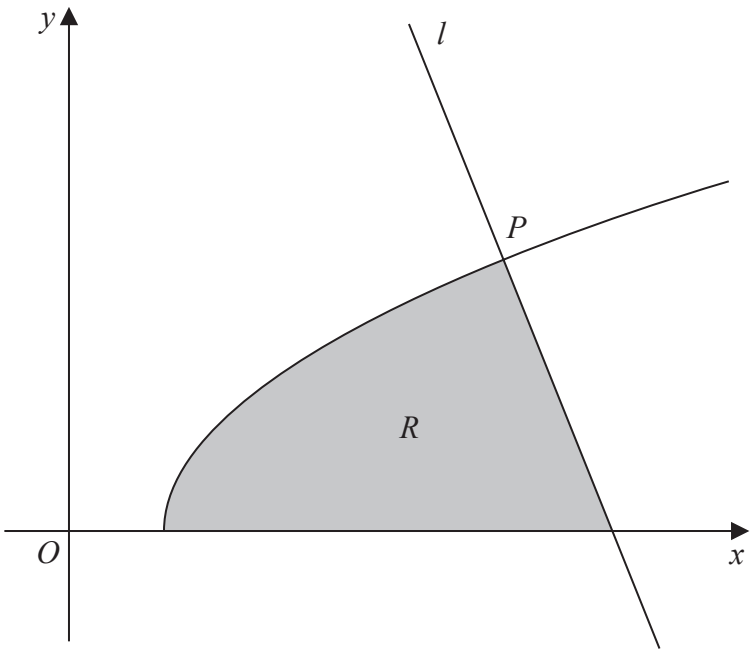



**Figure 3**

**In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

Figure 3 shows a sketch of part of the curve with equation

$$y = \sqrt{4x - 7}$$

The line $l$, shown in Figure 3, is the normal to the curve at the point $P(8, 5)$

(a) Use calculus to show that an equation of $l$ is

$$5x + 2y - 50 = 0$$

**(5)**

The region $R$, shown shaded in Figure 3, is bounded by the curve, the $x$-axis and $l$.

(b) Use algebraic integration to find the exact area of $R$.

**(4)**

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**Question 6 continued**

**Question 6 continued**

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**Question 6 continued**

**(Total for Question 6 is 9 marks)**

**7.** **In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

(a) Given that

$$\sqrt{2}\sin(x + 45°) = \cos(x - 60°)$$

show that

$$\tan x = -2 - \sqrt{3}$$

**(4)**

(b) Hence or otherwise, solve, for $0 \leqslant \theta < 180°$

$$\sqrt{2}\sin(2\theta) = \cos(2\theta - 105°)$$

**(4)**

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**Question 7 continued**

**Question 7 continued**

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**Question 7 continued**

**(Total for Question 7 is 8 marks)**

**8.**

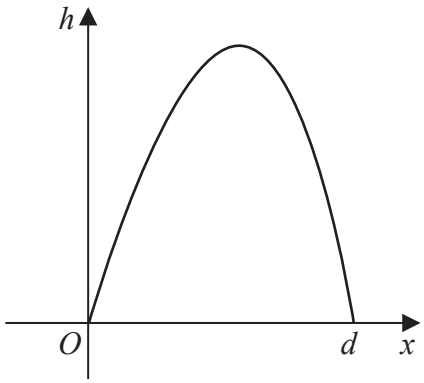



**Figure 4**

Figure 4 is a graph showing the path of a golf ball after the ball has been hit until it first hits the ground.

The vertical height, $h$ metres, of the ball above the ground has been plotted against the horizontal distance travelled, $x$ metres, measured from where the ball was hit.

The ball travels a horizontal distance of $d$ metres before it first hits the ground.

The ball is modelled as a particle travelling in a vertical plane above horizontal ground.

The path of the ball is modelled by the equation

$$h = 1.5x - 0.5x\,e^{0.02x} \qquad 0 \leqslant x \leqslant d$$

**Use the model to answer parts (a), (b) and (c).**

(a) Find the value of $d$, giving your answer to 2 decimal places.

(*Solutions relying entirely on calculator technology are not acceptable.*)

**(3)**

(b) Show that the maximum value of $h$ occurs when

$$x = 50\ln\left(\frac{150}{x + 50}\right)$$

**(4)**

Using the iteration formula

$$x_{n+1} = 50\ln\left(\frac{150}{x_n + 50}\right) \qquad \text{with } x_1 = 30$$

(c) (i) find the value of $x_2$ to 2 decimal places,

(ii) find, by repeated iteration, the horizontal distance travelled by the golf ball before it reaches its maximum height. Give your answer to 2 decimal places.

**(3)**

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**Question 8 continued**

**Question 8 continued**

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**Question 8 continued**

**(Total for Question 8 is 10 marks)**

**9.**

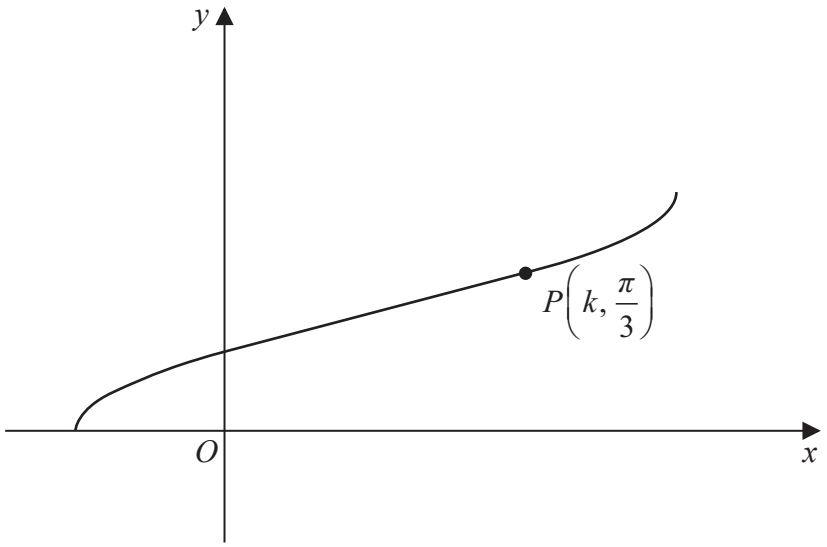



**Figure 5**

The curve shown in Figure 5 has equation

$$x = 4\sin^2 y - 1 \qquad 0 \leqslant y \leqslant \frac{\pi}{2}$$

The point $P\left(k, \frac{\pi}{3}\right)$ lies on the curve.

(a) Verify that $k = 2$

**(1)**

(b) (i) Find $\frac{\mathrm{d}x}{\mathrm{d}y}$ in terms of $y$

(ii) Hence show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2\sqrt{x+1}\sqrt{3-x}}$

**(6)**

The normal to the curve at $P$ cuts the $x$-axis at the point $N$.

(c) Find the exact area of triangle $OPN$, where $O$ is the origin.

Give your answer in the form $a\pi + b\pi^2$ where $a$ and $b$ are constants.

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**Question 9 continued**

**Question 9 continued**

**(Total for Question 9 is 10 marks)**

**TOTAL FOR PAPER IS 75 MARKS**

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