Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WMA13/01**

Mathematics

International Advanced Level
Pure Mathematics P3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

$$f(x) = \frac{2x^3 - 4x - 15}{x^2 + 3x + 4}$$

(a) Show that

$$f(x) \equiv Ax + B + \frac{C(2x + 3)}{x^2 + 3x + 4}$$

where A, B and C are integers to be found.

(4)

(b) Hence, find

$$\int_3^5 f(x)\,dx$$

giving your answer in the form $p + \ln q$, where p and q are integers.

(5)

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Question 1 continued

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Question 1 continued

Q1

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2. The functions f and g are defined by

$$f(x) = 5 - \frac{4}{3x + 2} \qquad x \geqslant 0$$

$$g(x) = \left| 4\sin\left(\frac{x}{3} + \frac{\pi}{6}\right) \right| \qquad x \in \mathbb{R}$$

(a) Find the range of f

(2)

(b) (i) Find $f^{-1}(x)$

(ii) Write down the domain of f^{-1}

(3)

(c) Find $fg(-\pi)$

(2)

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Question 2 continued

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3. **In this question you must show all stages of your working.**

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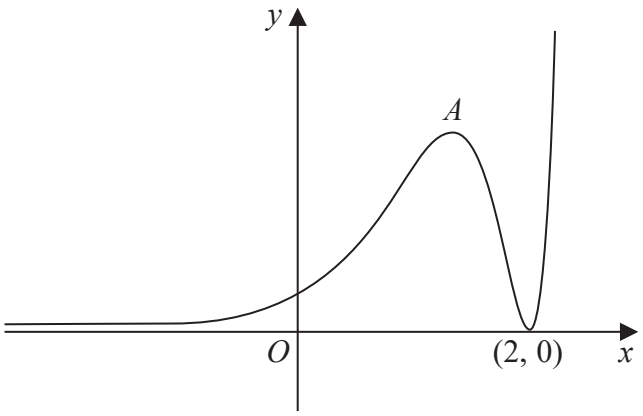


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = (x - 2)^2 \mathrm{e}^{3x} \qquad x \in \mathbb{R}$$

The curve has a maximum turning point at A and a minimum turning point at (2, 0)

(a) Use calculus to find the exact coordinates of A.

(5)

Given that the equation $\mathrm{f}(x) = k$, where k is a constant, has **at least** two distinct roots,

(b) state the range of possible values for k.

(2)

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Question 3 continued

Q3

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4. $y = \log_{10}(2x + 1)$

(a) Express x in terms of y.

(2)

(b) Hence, giving your answer in terms of x, find $\frac{dy}{dx}$

(3)

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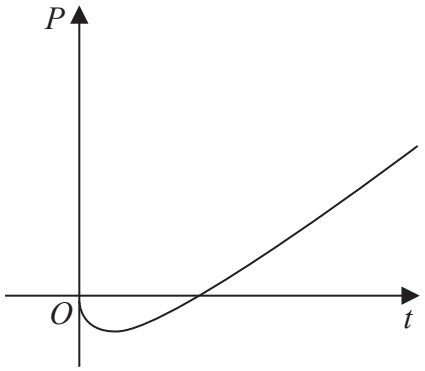


Figure 2

The profit made by a company, £P million, t years after the company started trading, is modelled by the equation

$$P = \frac{4t-1}{10} + \frac{3}{4}\ln\left[\frac{t+1}{(2t+1)^2}\right]$$

The graph of P against t is shown in Figure 2.

According to the model,

(a) show that exactly one year after it started trading, the company had made a loss of approximately £830 000

(2)

A manager of the company wants to know the value of t for which $P = 0$

(b) Show that this value of t occurs in the interval $[6, 7]$

(2)

(c) Show that the equation $P = 0$ can be expressed in the form

$$t = \frac{1}{4} + \frac{15}{8}\ln\left[\frac{(2t+1)^2}{t+1}\right]$$

(2)

(d) Using the iteration formula

$$t_{n+1} = \frac{1}{4} + \frac{15}{8}\ln\left[\frac{(2t_n+1)^2}{t_n+1}\right] \quad \text{with} \quad t_1 = 6$$

find the value of t_2 and the value of t_6, giving your answers to 3 decimal places.

(3)

(e) Hence find, according to the model, how many months it takes in total, from when the company started trading, for it to make a profit.

(2)

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Question 5 continued

Q5

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6.

$$y = \frac{2 + 3\sin x}{\cos x + \sin x}$$

Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{a\tan x + b\sec x + c}{\sec x + 2\sin x}$$

where a, b and c are integers to be found.

(6)

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Question 6 continued

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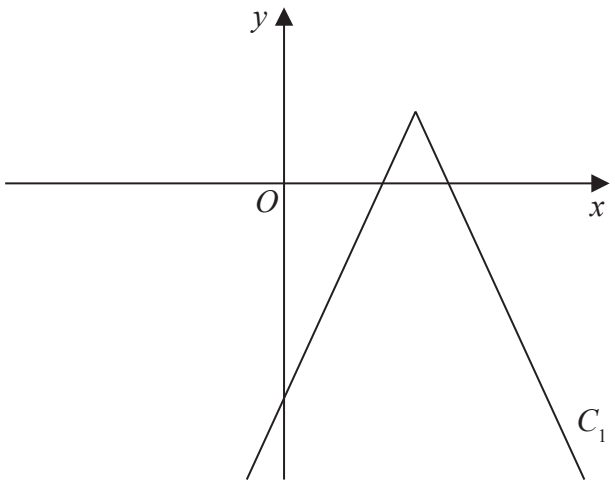


Figure 3

Figure 3 shows a sketch of the graph of C_1 with equation

$$y = 5 - |3x - 22|$$

(a) Write down the coordinates of

(i) the vertex of C_1

(ii) the intersection of C_1 with the y-axis.

(2)

(b) Find the x coordinates of the intersections of C_1 with the x-axis.

(2)

Diagram 1, shown on page 21, is a copy of Figure 3.

(c) On Diagram 1, sketch the curve C_2 with equation

$$y = \frac{1}{9}x^2 - 9$$

Identify clearly the coordinates of any points of intersection of C_2 with the coordinate axes.

(3)

(d) Find the coordinates of the points of intersection of C_1 and C_2
(*Solutions relying entirely on calculator technology are not acceptable.*)

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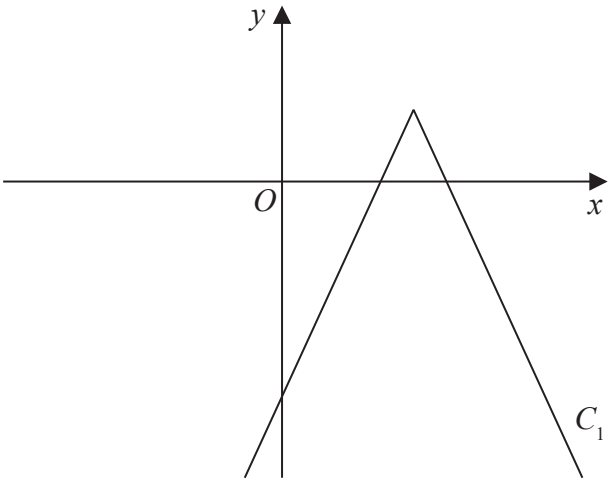


Diagram 1

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Question 7 continued

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8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Express $8\sin x - 15\cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R, and give the value of α, in radians, to 4 significant figures.

(3)

$$f(x) = \frac{15}{41 + 16\sin x - 30\cos x} \qquad x > 0$$

(b) Find

(i) the minimum value of $f(x)$

(ii) the smallest value of x at which this minimum value occurs.

(4)

(c) State the y coordinate of the minimum points on the curve with equation

$$y = 2f(x) - 5 \qquad x > 0$$

(1)

(d) State the smallest value of x at which a maximum point occurs for the curve with equation

$$y = -f(2x) \qquad x > 0$$

(1)

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9. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Given that $\cos 2\theta - \sin 3\theta \neq 0$

(a) prove that

$$\frac{\cos^2\theta}{\cos 2\theta - \sin 3\theta} \equiv \frac{1 + \sin\theta}{1 - 2\sin\theta - 4\sin^2\theta}$$

(4)

(b) Hence solve, for $0 < \theta \leqslant 360°$

$$\frac{\cos^2\theta}{\cos 2\theta - \sin 3\theta} = 2\operatorname{cosec}\theta$$

Give your answers to one decimal place.

(5)

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TOTAL FOR PAPER IS 75 MARKS

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