Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WFM03/01**

Mathematics

International Advanced Subsidiary/ Advanced Level
Further Pure Mathematics F3

You must have:
Mathematical Formulae and Statistics Tables (Yellow), cal

Candidates may use any calculator allowed by Pearson re
must not have the facility for symbolic algebra manipulat
integration, or have retrievable mathematical formulae st

NOTE:

This is the original version of WFM03_01, sat by candidates. Question 5c cannot be answered, as **A** is not a symmetric matrix.

Appropriate actions have been taken to ensure candidates who sat this version have not been negatively impacted by this error.

Please do not use this version of the Question Paper for mocks and future student assessments.
An updated version of the question paper will be released to be used for mocks and class assessments.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of q
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

P72468A

©2022 Pearson Education Ltd.
J:1/1/1/1/1/

P 7 2 4 6 8 A 0 1 3 2

Pearson

1. Given that

$$y = 3x \arcsin 2x \qquad 0 \leqslant x \leqslant \frac{1}{2}$$

(a) determine an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$

(2)

(b) Hence determine the exact value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x = \frac{1}{4}$, giving your answer in the form $a\pi + b$ where a and b are fully simplified constants to be found.

(1)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 1 continued

(Total for Question 1 is 3 marks)

2. A hyperbola H has equation

$$\frac{x^2}{a^2} - \frac{y^2}{5} = 1 \quad \text{where } a \text{ is a positive constant}$$

The line with equation $x = \frac{4}{3}$ is a directrix of H

(a) Write down an equation of the other directrix.

(1)

(b) Determine

(i) the value of a

(ii) the coordinates of each of the foci of H

(5)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 2 continued

(Total for Question 2 is 6 marks)

3. Solve the equation

$$4\tanh x - \operatorname{sech} x = 1$$

giving your answer in the form $x = \ln k$ where k is a fully simplified rational number.

(6)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 3 continued

(Total for Question 3 is 6 marks)

4. (a) Determine

$$\int \frac{1}{\sqrt{9x^2 + 16}}\,\mathrm{d}x$$

(2)

(b) Hence determine the exact value of

$$\int_{-2}^{2} \frac{1}{\sqrt{9x^2 + 16}}\,\mathrm{d}x$$

Give your answer in the form $a\ln(b + c\sqrt{13})$, where a, b and c are rational numbers.

(3)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 4 continued

Question 4 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 4 continued

(Total for Question 4 is 5 marks)

5.

$$\mathbf{A} = \begin{pmatrix} a & a & 1 \\ -a & 4 & 0 \\ 4 & a & 5 \end{pmatrix}$$ where a is a positive constant

Given that 2 is an eigenvalue of $\mathbf{A}$

(a) determine

(i) the value of a

(ii) the other two eigenvalues of $\mathbf{A}$

(5)

A normalised eigenvector for the eigenvalue 2 is $\begin{pmatrix} \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ -\frac{2}{\sqrt{6}} \end{pmatrix}$

(b) Determine a normalised eigenvector for each of the other eigenvalues of $\mathbf{A}$

(5)

(c) Hence determine a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{P}^{\mathrm{T}}\mathbf{AP} = \mathbf{D}$

(2)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 5 continued

Question 5 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 5 continued

(Total for Question 5 is 12 marks)

6. A curve has parametric equations

$$x = a(\theta - \sin\theta)$$
$$y = a(1 - \cos\theta)$$

where a is a positive constant.

(a) Show that

$$\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = ka^2 \sin^2\frac{\theta}{2}$$

where k is a constant to be determined.

(4)

The part of the curve from $\theta = 0$ to $\theta = 2\pi$ is rotated through 2π radians about the x-axis.

(b) Determine the area of the surface generated, giving your answer in terms of π and a.

[*Solutions relying on calculator technology are not acceptable.*]

(5)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 6 continued

Question 6 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 6 continued

(Total for Question 6 is 9 marks)

7. The plane Π has equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 3 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

where λ and μ are scalar parameters.

(a) Determine a vector perpendicular to Π

(2)

The line l meets Π at the point (1, 2, 3) and passes through the point (1, 0, 1)

(b) Determine the size of the acute angle between Π and l

Give your answer to the nearest degree.

(4)

(c) Determine the shortest distance between Π and the point (6, −3, −6)

(4)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 7 continued

Question 7 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 7 continued

(Total for Question 7 is 10 marks)

8.

$$I_n = \int \cos^n x \, dx \qquad n \geqslant 0$$

(a) Prove that for $n \geqslant 2$

$$I_n = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n} I_{n-2}$$

(4)

(b) Show that for positive even integers n

$$\int_0^{\frac{\pi}{2}} \cos^n x \, dx = \frac{(n-1)(n-3)...5 \times 3 \times 1}{n(n-2)(n-4)...6 \times 4 \times 2} \times \frac{\pi}{2}$$

(4)

(c) Hence determine the exact value of

$$\int_0^{\frac{\pi}{2}} \cos^6 x \sin^2 x \, dx$$

(3)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 8 continued

Question 8 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 8 continued

(Total for Question 8 is 11 marks)

9. The ellipse E has equation

$$x^2 + 9y^2 = 9$$

The foci of E are F_1 and F_2

(a) (i) Determine the coordinates of F_1 and the coordinates of F_2

(ii) Write down the equation of each of the directrices of E

(4)

The point P lies on the ellipse.

(b) Show that $|PF_1| + |PF_2| = 6$

(3)

The straight line through P with equation $y = 2x + c$ meets E again at the point Q

The point M is the midpoint of PQ

(c) Show that as P varies the locus of M is a straight line passing through the origin.

(6)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 9 continued

Question 9 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 9 continued

Question 9 continued

(Total for Question 9 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA