Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Thursday 15 June 2023

Morning (Time: 1 hour 30 minutes)

Paper reference **WFM03/01**

Mathematics

International Advanced Subsidiary/Advanced Level

Further Pure Mathematics F3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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1. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Solve the equation

$$7\cosh x + 3\sinh x = 2e^x + 7$$

Give your answers as simplified natural logarithms.

(5)

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Question 1 continued

(Total for Question 1 is 5 marks)

2. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

$$\mathbf{M} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 4 \\ 3 & -2 & -3 \end{pmatrix}$$

(a) Determine $\mathbf{M}^{-1}$

(3)

The transformation represented by **M** maps the plane Π_1 to the plane Π_2

The point (x, y, z) on Π_1 maps to the point (u, v, w) on Π_2

(b) Determine x, y and z in terms of u, v and w as appropriate.

(3)

The plane Π_1 has equation

$$3x - 7y + 2z = -3$$

(c) Find a Cartesian equation for Π_2

Give your answer in the form $au + bv + cw = d$ where a, b, c and d are integers to be determined.

(2)

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Question 2 continued

Question 2 continued

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Question 2 continued

(Total for Question 2 is 8 marks)

3.

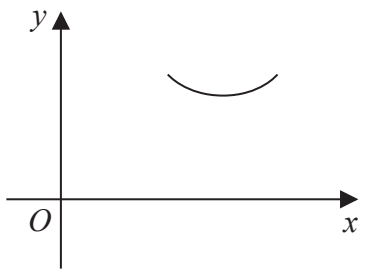


Figure 1

Figure 1 shows a sketch of the curve C with equation

$$y = \frac{1}{2}\left(\tan x + \cot x\right) \qquad \frac{\pi}{6} \leqslant x \leqslant \frac{\pi}{3}$$

(a) Show that the length of C is given by

$$\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\left(\tan^2 x + \cot^2 x\right)\mathrm{d}x$$

(6)

(b) Hence determine the exact length of C, giving your answer in simplest form.

(5)

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Question 3 continued

Question 3 continued

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Question 3 continued

(Total for Question 3 is 11 marks)

4. The plane Π_1 contains the point $A(2, 4, -5)$ and is normal to the vector $\begin{pmatrix} -1 \\ 3 \\ 3 \end{pmatrix}$

The plane Π_2 contains the point $B(3, 6, -2)$ and is normal to the vector $\begin{pmatrix} 2 \\ 0 \\ -5 \end{pmatrix}$

The line l is the line of intersection of Π_1 and Π_2

(a) Determine a vector equation for l.

(7)

The points C and D both lie on l.

Given that C and D are 5 units apart,

(b) determine the exact volume of the tetrahedron $ABCD$.

(5)

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Question 4 continued

Question 4 continued

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Question 4 continued

(Total for Question 4 is 12 marks)

5.

$$\mathbf{M} = \begin{pmatrix} 1 & 2 & k \\ -1 & -3 & 4 \\ 2 & 6 & -8 \end{pmatrix} \quad \text{where } k \text{ is a constant}$$

Given that **M** has a repeated eigenvalue, determine

(i) the possible values of k,

(ii) all corresponding eigenvalues of **M** for each value of k.

(7)

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Question 5 continued

Question 5 continued

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Question 5 continued

(Total for Question 5 is 7 marks)

6. The ellipse E has equation $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$

The point $P(4\cos\theta, 3\sin\theta)$ lies on E.

(a) Use calculus to show that an equation of the tangent to E at P is

$$3x\cos\theta + 4y\sin\theta = 12$$

(5)

(b) Determine an equation for the normal to E at P.

(2)

The tangent to E at P meets the x-axis at the point A.
The normal to E at P meets the y-axis at the point B.

(c) Show that the locus of the midpoint of A and B as θ varies has equation

$$x^2(p - qy^2) = r$$

where p, q and r are integers to be determined.

(6)

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Question 6 continued

Question 6 continued

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Question 6 continued

(Total for Question 6 is 13 marks)

7. $$I_n = \int \cosh^n 2x \, \mathrm{d}x \qquad n \geqslant 0$$

(a) Show that, for $n \geqslant 2$

$$I_n = \frac{\cosh^{n-1} 2x \sinh 2x}{2n} + \frac{n-1}{n} I_{n-2}$$

(5)

(b) Hence determine

$$\int (1 + \cosh 2x)^3 \, \mathrm{d}x$$

collecting any like terms in your answer.

(4)

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Question 7 continued

Question 7 continued

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Question 7 continued

(Total for Question 7 is 9 marks)

8. (a) Differentiate $x\operatorname{arcosh} 5x$ with respect to x

(2)

(b) Hence, or otherwise, show that

$$\int_{\frac{1}{4}}^{\frac{3}{5}} \operatorname{arcosh} 5x \, \mathrm{d}x = \frac{3}{20} - \frac{2\sqrt{2}}{5} + \ln\left(p + q\sqrt{2}\right)^k - \frac{1}{4}\ln r$$

where p, q, r and k are rational numbers to be determined.

(8)

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Question 8 continued

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Question 8 continued

Question 8 continued

(Total for Question 8 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS

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