GradeMax



Mark Scheme (Results)

Summer 2019

Pearson Edexcel GCE In Decision Mathematics D1 Paper 6689/01

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Summer 2019
Publications Code 6689_01_1906_MS
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- M marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- B marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark ⌙

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer

Question Number	Scheme	Marks
1. (a)	Bipartite (graph)	B1 **(1)**
(b)	e.g. (see below for alternatives)	
	First alternating path: B – 6 = E – 5 = C – 4	M1
	Change status to give: B = 6 – E = 5 – C = 4	A1
	Improved matching: A = 3, B = 6, C = 4, (D unmatched), E = 5, F = 1	A1
	Second alternating path: D – 3 = A – 5 = E – 1= F – 2	M1
	Change status to give: D = 3 – A = 5 – E = 1 – F = 2	A1
	Complete matching: A = 5, B = 6, C = 4, D = 3, E = 1, F = 2	A1 **(6)**
		7 marks

Notes for Question 1

Possible 1st paths	A	B	C	D	E	F	Subsequent 2nd paths
B – 6 – E – 5 – C – 4	3	6	4	-	5	1	D – 3 – A – 5 – E – 1 – F – 2
B – 6 – E – 1 – F – 2	3	6	5	-	1	2	D – 3 – A – 5 – C – 4
D – 3 – A – 5 – C – 4	5	-	4	3	6	1	B – 6 – E – 1 – F – 2

a1B1: CAO - but be charitable on spelling, award if phonetically close

b1M1: An alternating path (e.g. letter 1st set – number 2nd set – letter 1st set – …) from B to 4 (or vice-versa) or B to 2 (or vice-versa) or D to 4 (or vice-versa)
b1A1: CAO – a correct path including change status **either** stated (only accept 'change (of) status' **or** 'c.s' but not, e.g.'change state') **or** shown (all symbols e.g. (…– … = … – …) interchanged (… = …. – … = …)) **chosen path clear**
b2A1: CAO – improved matching - must follow from the correct stated path. Accept either stated **or** on a **clear** diagram (with five arcs **only**). **Please check the top of the second page as many candidates will draw either the improved or complete matching on the nodes provided there**
b2M1: An alternating path from D to 2 or D to 4 or B to 2 (or vice-versa in each case) (**must** be consistent with their first alternating path)
b3A1: CAO – a correct path including change status stated **or** shown. **Chosen path clear**
b4A1: CAO (complete matching) must follow from two correct stated paths (so **both** previous M marks must have been awarded). Accept on a clear diagram (with six arcs **only**)

SC for (b): some candidates are writing AP2 and AP3 from the first list next to each other with no improved matching and so it is not clear if they are applying the algorithm correctly e.g.
alternating path:
B – 6 = E – 1 = F – 2; D – 3 = A – 5 = C = 4
Change status:
B = 6 – E = 1 – F = 2; D = 3 – A = 5 – C = 4
Complete matching
A = 5, B = 6, C = 4, D = 3, E = 1, F = 2

This would scored M1 A0 A0 M1 A0 A1 (neither chosen path is clear – are they choosing two APs from the first list or does one follow the other (reading across the page) – either way it is not clear)

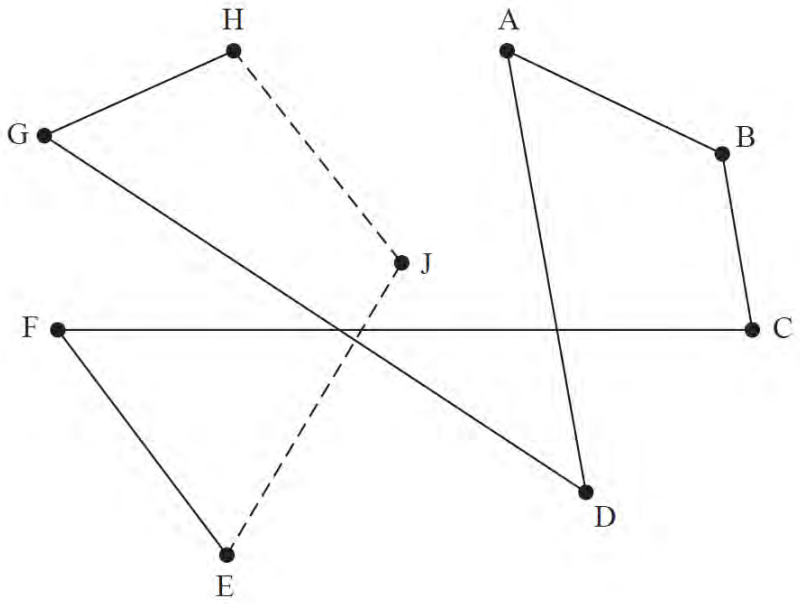
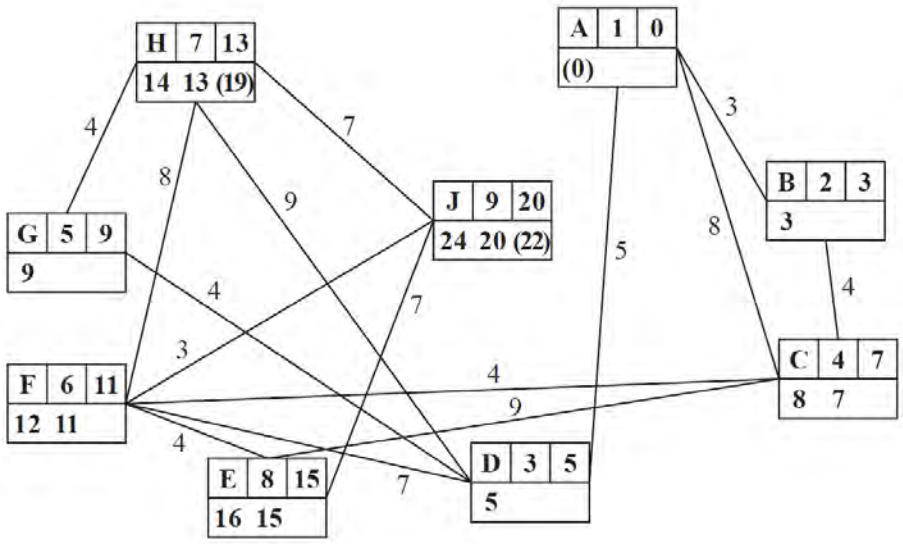
Question Number	Scheme	Marks
2.(a)	Pair the two odds nodes: A with H	B1
	Repeat arcs AG, GJ, JH	B1
	Length = 48.2 + (2.5 + 0.9 + 1.1) = 48.2 + 4.5 = 52.7	B1
	e.g. route FAGABFGJGHJHFEJBCDECF	B1 **(4)**
(b)	AD + FH = A(BC)D + F(GJ)H = 6 + 3.7 = 9.7	M1 A1
	AF + DH = AF + D(EJ)H = 2.4 + 5.5 = 7.9*	A1
	AH + DF = A(GJ)H + D(E)F = 4.5 + 3.8 = 8.3	A1
	Repeat arcs AF, DE, EJ and JH	A1 **(5)**
(c)	Route from D to F is longer	B1
	Difference = (48.2 + 7.9) – 52.7 = 3.4 or 7.9 – 4.5 = 3.4	B1 **(2)**
		11 marks

Notes for Question 2

a1B1: CAO (correct pairing of the two odd nodes stated) – this mark may be implied by consideration of the shortest path from A to H (or vice-versa)
a2B1: Correct repeated arcs stated (AG, GJ, JH)
a3B1: CAO (length of 52.7)
a4B1: Correct route: checks – starts and finishes at F, 21 nodes, AG, GJ and JH repeated, A(2), B(2), C(2), D(1), E(2), F(4), G(3), H(2), J(3)

b1M1: Three distinct pairings of nodes A, D, F and H
b1A1: Any one row correct including pairing **and** total
b2A1: Any two rows correct including pairings **and** totals
b3A1: All three rows correct including pairings **and** totals
b4A1: CAO correct arcs clearly (not just in their working) stated: AF, DE, EJ and JH. Do not accept DH, DEJH or DH via E and J

c1B1: CAO (oe e.g. F to F is shorter) – dependent on correct repeats in (a) and (b) **or** clearly implied in (c) (e.g. correct values compared in this part)
c2B1: CAO (difference of 3.4 **or** comparing 52.7 and 56.1 **or** comparing 7.9 with 4.5)

Question Number	Scheme	Marks
3.(a)	Prim: AB, BC, CF, EF; AD, DG; GH, EJ or HJ	M1; A1; A1
	Weight of tree = 35 (metres)	A1
		A1 **(5)**
(b)	No – there are two different MST (with a weight of 35) either with EJ or HJ	B1 **(1)**
(c)		M1 A1 (BDCG) A1 (FH) A1 (EJ)
	Length of shortest path is 20 (metres)	A1ft **(5)**
		11 marks

Notes for Question 3

a1M1: First four arcs correctly chosen in order {AB, BC, CF, EF…} or first five nodes correctly chosen in order{A, B, C, F, E,…}. If any rejections seen at any point then M1 (max) only. Order of nodes may be seen at the top of the matrix {1, 2, 3, -, 5, 4, -, -, -} so please check the top of the matrix carefully
a1A1: First six arcs correctly chosen in order {AB, BC, CF, EF, AD, DG, …}or all nine nodes correctly chosen in order{A, B, C, F, E, D, G, H, J}. Order of nodes may be seen at the top of the matrix so for the first two marks accept {1, 2, 3, 6, 5, 4, 7, 8, 9} (do not condone any missing numbers e.g. the number 9 must be above J)
a2A1: CSO – all arcs correct stated and chosen in the correct order. Candidates must be considering arcs for this final mark (do not accept a list of nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen). Allow EJ or HJ for the final arc but not EJ **and** HJ
a3A1: CAO (weight of 35)
a4A1: CAO (do not award this mark if both EJ and HJ are shown in the MST)

b1B1: CAO – no (oe) + mention of two different MST with either arc EJ or HJ – as a minimum must mention EJ and HJ – or clearly implied that the final arc in the MST could be replaced with the other arc (e.g. if final arc in MST is EJ they must then say that HJ could replace the final arc in the MST)
(do not award this mark if the candidate believes that unique means that there is more than one MST)

In (c) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded and with no additional working values. It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling

c1M1: A larger value replaced by a smaller value at least once in the working values at either C or E or F or H or J
c1A1: All values at B, D, C, G correct and the working values in the correct order at C
c2A1: All values at F and H correct and the working values in the correct order at F and H
c3A1: All values at E and J correct and the working values in the correct order at E and J
c4A1ft: Follow through the value at J only

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>4.(a)</td><td>$\frac{204}{50} = 4.08$ so lower bound is 5 containers</td><td>M1 A1 (2)</td></tr>
<tr><td>(b)</td><td>Container 1: 25 9 16
Container 2: 32 17
Container 3: 23 18 4
Container 4: 12 8
Container 5: 40</td><td>M1 A1 (2)</td></tr>
<tr><td>(c)</td><td>middle right
<table><tr><td>25</td><td>9</td><td>32</td><td>16</td><td>17</td><td>23</td><td>18</td><td>12</td><td>4</td><td>8</td><td>40</td></tr><tr><td>25</td><td>32</td><td>40</td><td>23</td><td>9</td><td>16</td><td>17</td><td>18</td><td>12</td><td>4</td><td>8</td></tr><tr><td>40</td><td>32</td><td>25</td><td>23</td><td>18</td><td>9</td><td>16</td><td>17</td><td>12</td><td>4</td><td>8</td></tr><tr><td>40</td><td>32</td><td>25</td><td>23</td><td>18</td><td>16</td><td>17</td><td>12</td><td>9</td><td>4</td><td>8</td></tr><tr><td>40</td><td>32</td><td>25</td><td>23</td><td>18</td><td>17</td><td>16</td><td>12</td><td>9</td><td>8</td><td>4</td></tr><tr><td>40</td><td>32</td><td>25</td><td>23</td><td>18</td><td>17</td><td>16</td><td>12</td><td>9</td><td>8</td><td>4</td></tr></table>middle left
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A1
A1ft
A1 (4)</td></tr>
<tr><td>(d)</td><td>Container 1: 40 9
Container 2: 32 18
Container 3: 25 23
Container 4: 17 16 12 4
Container 5: 8</td><td>M1 A1 A1 (3)</td></tr>
<tr><td>(e)</td><td>Total weight of suitcases: $132 + 2x$</td><td>B1</td></tr>
<tr><td></td><td>$3 < \frac{132 + 2x}{50} \leq 4$</td><td>M1</td></tr>
<tr><td></td><td>$9 < x \leq 34$</td><td>A1 A1 (4)</td></tr>
<tr><td></td><td></td><td>15 marks</td></tr>
</table>

GradeMax

Notes for Question 4

PLEASE NOTE NO MISREADS IN THIS QUESTION – MARK ACCORDING TO THE SCHEME AND THE SPECIAL CASES IN PARTS (c) AND (d)

a1M1: Attempt to find the lower bound (204 ± 40) / 50 (a value of 4.08 seen with no working can imply this mark)
a1A1: CSO - correct calculation seen or 4.08 followed by 5 (containers) – accept 4.1 followed by 5 if correct calculation seen. An answer of 5 with no working scores M0A0

b1M1: First five items placed correctly and at least eight values placed in containers. Condone cumulative totals for M1 only (the values in bold)
b1A1: CSO (correct solution only – so no additional/repeated values)

c1M1: Quick sort – pivot, p, chosen (must be choosing middle left or middle right – choosing first/last item as pivot is M0). After the first pass the list must read (values greater than the pivot), pivot, (values less that the pivot). If only choosing one pivot per iteration then M1 only – bubble sort is not a MR and scores M1 only for 25 32 16 17 23 18 12 9 8 40 4
c1A1: First two passes correct and next pivots chosen correctly for third pass (but third pass does not need to be correct) – so they must be choosing (if middle right) pivot value of 12 or (if middle left) pivot value of 17
c2A1ft: Third and fourth passes correct (follow through from their second pass and choice of pivots). They do not need to be choosing a pivot for the fifth pass for this mark
c3A1: CSO (correct solution only – all previous marks in this part must have been awarded) - if middle right then a fifth pass in which the 8 is used as a pivot must be included or if middle left then a fifth pass in which the 4 is used as a pivot and a sixth pass in which the 9 is used as a pivot must be included

SC for (c): If using an incorrect list from the start of (c) with only one error (an error is either one missing number, one extra number, one incorrect number or one number incorrectly placed) then the most they can score is M1A0A1ftA0

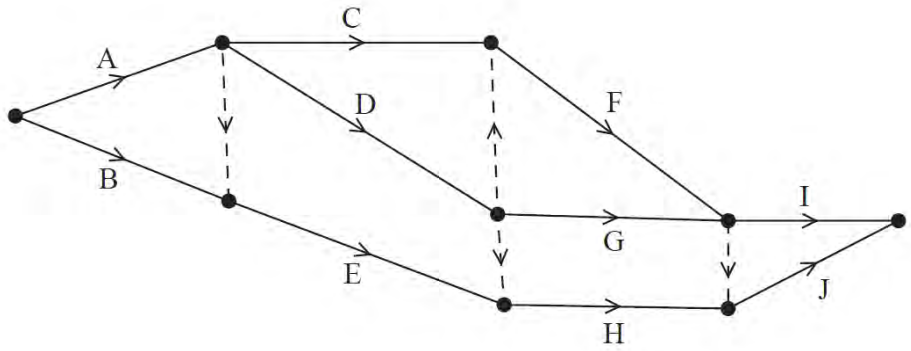
Sorting list into ascending order in (c)

- If the candidate sorts the list into ascending order and reverses the list in this part then this can score full marks in (c)
- If the list is not reversed in (c) then remove the last two A marks earned in (c). If the list is reversed at the start of (d) but not in (c) then still remove the last two A marks earned in (c). If the list is in ascending order in (c) award no marks for first-fit increasing in (d). If the candidate says that the list needs reversing in (c) but does not actually show the reversed list in (c) then remove the last A mark earned

d1M1: Must be using the correct sorted list in descending order. First five items placed correctly and at least eight values placed in bins – condone cumulative totals for M1 only (the bold values)
d1A1: First seven items placed correctly (the underlined and bold values)
d2A1: CSO (so no additional/repeated values)

SC for (d) – if 'sorted' list is incorrect from part (c) and M0 would be awarded in (d) then award M1 only in (d) for their first eight items correctly placed – by 'incorrect' they can have only one 'error'- an 'error' is one missing number, one extra number, one incorrect number or one number incorrectly placed. Allow full marks in (d) if a correct list is used in (d) even if the list is incorrect at the end of (c). Please note that if 'sorted' list is incorrect in (c) and it is clear that this has been used from their working in (d) then please award at most M1 in (d)

e1B1: $132 + 2x$ (oe)
e1M1: $((132 \pm 15) + 2x) / 50$ 'equated' to their 3 or their 4 (that is 1 or 2 less than the lower bound stated in (a))
e1A1: Either 9 or 34 correctly found
e2A1: $9 < x \leq 34$ as a final answer (do not isw incorrect 'simplification')

Question Number	Scheme	Marks
5. (a)		M1 A1 A1 A1 A1 **(5)**
(b)	In the network there must be at least one critical path from the start node to the end node and as all paths must pass through either I or J (as these are the only two activities that go into the end node) so one of these must be critical	B1 **(1)**
(c)	A and F must be critical	B1 **(1)**
		7 marks

Notes for Question 5

In (a) condone lack of, or incorrect, numbered events throughout. 'Dealt with correctly' means that the activity starts from the correct event but need not necessarily finishes at the correct event, e.g. 'F dealt with correctly' requires the correct precedences for this activity, i.e. C and D labelled correctly and leading into the same node and F starting from that node but do not consider the end node for F. **Activity on node is M0**

If an arc is not labelled, for example, if the arc for activity G is not labelled (but the arc is present) then this will lose the second A mark and the final (CSO) A mark – they can still earn the third A mark on the bod. If two or more arcs are not labelled then mark according to the scheme. Assume that a solid line is an activity which has not been labelled rather than a dummy (even if in the corrct place for where a dummy should be)

Ignore incorrect or lack of arrows on the activities for the first four marks only

a1M1: Eight activities (labelled on arc), one start and at least two dummies placed
a1A1: Activities A, B, 1st dummy (+ arrow) and C, D and E dealt with correctly
a2A1: 2nd and 3rd dummies (+ arrow) and F, G and H dealt with correctly
a3A1: Activities I, J and 4th dummy (+ arrow) dealt with correctly
a4A1: CSO – all arrows present and correctly placed with one finish **– please check all arcs carefully for arrows**

Note that additional (but unnecessary) 'correct' dummies that still maintain precedence for the network should only be penalised with the final A mark if earned

b1B1: CAO (must either mention that any critical path for the network would need to pass through either I or J

c1B1: CAO (with no additional activities – so accept A, F **or** A, C, F **only** but not A, (C), F, I or J)

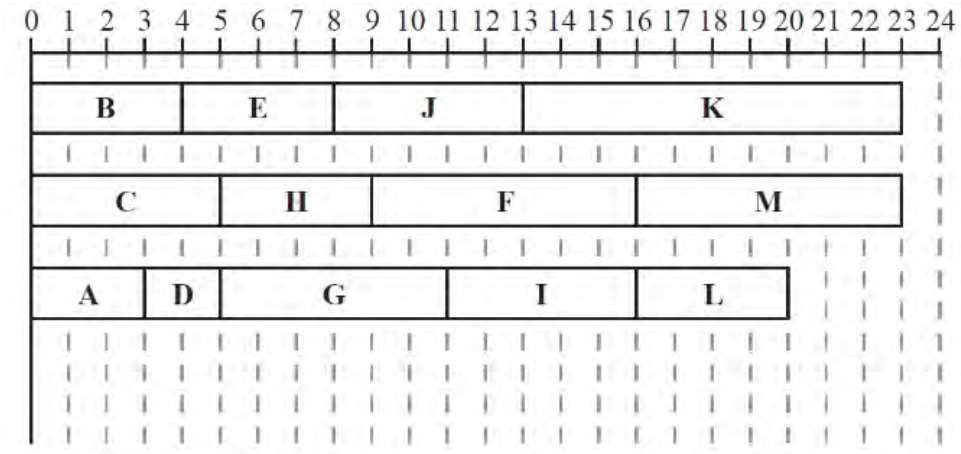
Question Number	Scheme	Marks
6.	$A(4,7), B(5,3), C(-1,5), D(-2,1)$	
(a)	Equation through AC e.g. $\frac{y-7}{5-7}=\frac{x-4}{-1-4}$ or $y-7=\left(\frac{5-7}{-1-4}\right)(x-4)$ or $y-5=\left(\frac{7-5}{4-(-1)}\right)(x+1)$	M1
	$5y-2x=27$ (oe)	A1
	$5y-2x\leq 27$	A1 **(3)**
(b)	Point testing A and B or objective line (with gradient of –5)	M1
	At $B(5, 3)$, $P = 28$ at $A(4, 7)$, $P = 27$ so optimal point is B with $P = 28$	A1 A1 **(3)**
(c)	$A > B \Rightarrow 4k + 7 > 5k + 3$ or objective line argument	M1
	$k < 4$	A1
	$A > C \Rightarrow 4k + 7 > -k + 5$ or objective line argument	M1
	$k > -\frac{2}{5}$	A1 **(4)**
		10 marks

Notes for Question 6

a1M1: Correct method for finding the equation of the line through A and C – a correct equation can imply this mark – condone one sign error **only**
a1A1: Correct equation (any correct form (allow unsimplified or simplified incorrectly) – condone any inequality sign or equals)
a2A1: CAO (any equivalent form provided coefficients are integers)

b1M1: Correct objective line drawn (gradient of –5 - acceptable minimum length is from $(0,1)$ to $(0.2,0)$) or testing both A **and** B in the correct objective function
b1A1: CAO (B or (5, 3))
b2A1: $P = 28$ (allow if seen in working for B)

c1M1: Put expression for A > expression for B (accept any inequality or equals) or considers gradient of line segment through A and B with $-k$. Condone x for k for the M mark only
c1A1: $k < 4$
c2M1: Put expression for A > expression for C (accept any inequality or equals) or considers gradient of line segment through A and C with $-k$. Condone x for k for the M mark only
c2A1: $k > -\frac{2}{5}$
Ignore any consideration of vertex D

Question Number	Scheme	Marks
7.(a)	$w = 7,\ x = 8,\ y = 6,\ z = 7$	B4,3,2,1,0 **(4)**
(b)	The dummy is required as J relies on D and E but M relies on D, E, F and I	B1 **(1)**
(c)	Critical activities: B, E, J and K	B1 **(1)**
(d)	e.g. $\frac{53+y+z}{23} = \frac{66}{23} = 2.869...$ so at least three workers are required	B1 **(1)**
(e)	e.g.  	M1 A1 A1 **(3)**
(f)	$35\times\left(\sum \text{activities completed by additional worker(s)}\right)$	M1
	£700	A1 **(2)**
(g)	H requiring 7 hours will delay the completion of the project because the total float on activity H is 2 hours and so the project will be delayed by 1 hour	M1 A1 **(2)**
		14 marks

Notes for Question 7

a1B1: One value correct
a2B1: Two values correct
a3B1: Three values correct
a4B1: All four values correct (check carefully for answers written on the diagram rather than on the given answer lines)

b1B1: Correct answer regarding precedence – must mention J, M and one of D and E and one of F and I

c1B1: CAO (B, E, J, K)

d1B1: Correct calculation or argument with the correct values of *y* and *z*. Other equivalent answers with regards to scheduling are acceptable (e.g. with only two workers the minimum completion time for the project is 33 hours **or** at time 2.5 activities A, B and C must be taking place (in situations like this detail of both activities and time must be given))

e1M1: Not a cascade chart. 4 ‘workers’ used at most and at least 9 activities placed
e1A1: 3 workers. All 13 activities present (just once). Condone at most two errors. An activity can give rise to at most three errors; one on duration, one on time interval and only one on IPA
e2A1: 3 workers. All 13 activities present (just once). No errors

Activity	Duration	Time interval	IPA
A	3	0 - 4	-
B	4	0 – 4	-
C	5	0 – 7	-
D	2	3 – 8	A
E	4	4 – 8	A, B
F	7	4 – 16	A, B
G	6	4 – 11	A, B
H	4	5 – 11	C
I	5	10 – 16	G, H
J	5	8 – 13	D, E
K	10	13 – 23	J
L	4	13 – 23	J
M	7	15 – 23	D, E, F, I

f1M1: Correct calculation (so cost of one or more additional workers only) for their schedule – dependent on scheduling at least 12 activities in (e) – M0 if attempted cost of the two permanent workers is included
f1A1: CAO (their schedule must have had two workers working continuously from 0 to 23) – condone lack of units (but not incorrect units)

g1M1: Delayed together with some mention of time and/or float for activity H
g1A1: Project delayed by 1 hour (oe e.g. minimum completion time is now 24) - just mentioning that the total float for activity H is 2 (or that H cannot be completed on time) is A0. Give bod that ‘a delay of 1 hour’ is considering the entire project but A0 if clearly only talking about activities

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