GradeMax

MODEL ANSWERS

Pearson Edexcel International Advanced Level

Decision Mathematics D1

Advanced/Advanced Subsidiary

Thursday 18 January 2018 – Afternoon
Time: 1 hour 30 minutes

Paper Reference
WDM01/01

You must have:
D1 Answer Book

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** on the top of the answer book with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the D1 answer book provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.
- Do not return the question paper with the answer book.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

P51509A

©2018 Pearson Education Ltd.

1/1/1/1/

P 5 1 5 0 9 A

Pearson

Write your answers in the D1 answer book for this paper.

1.

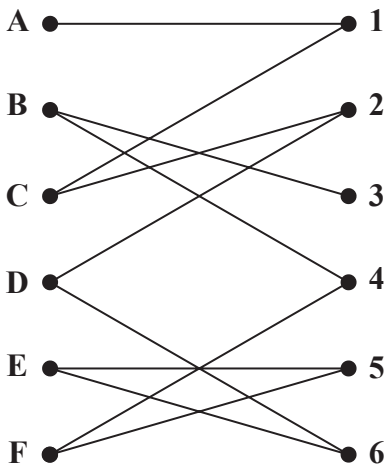


Figure 1

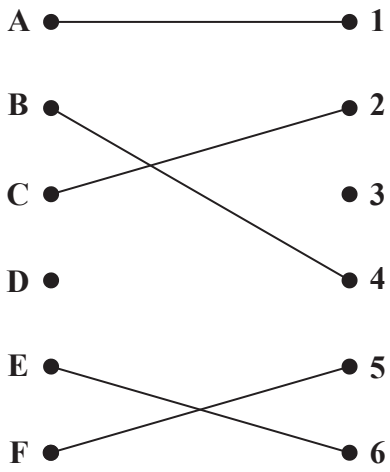


Figure 2

(a) Define the term ‘bipartite graph’.

(2)

Figure 1 shows the possible allocations of six people, A, B, C, D, E and F, to six activities, 1, 2, 3, 4, 5 and 6

Figure 2 shows an initial matching.

(b) Starting from this initial matching, use the maximum matching algorithm to find a complete matching. You should list the alternating path that you use, and state your complete matching.

(3)

(Total 5 marks)

Leave blank

1.

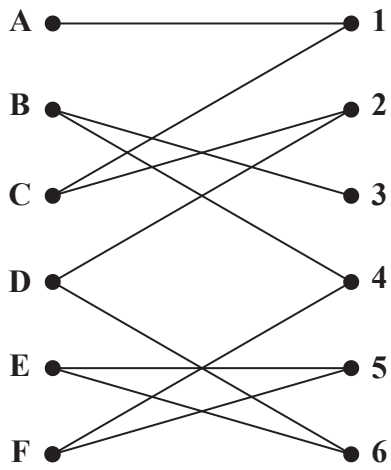


Figure 1

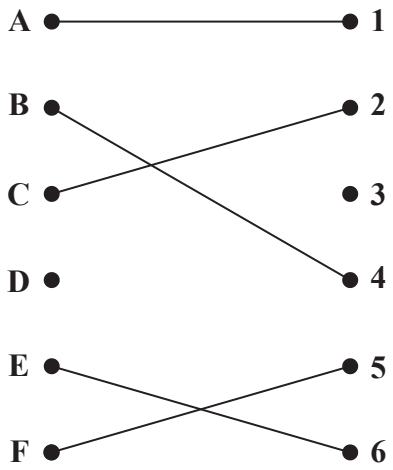


Figure 2

a) A bipartite graph has 2 sets of nodes, X and Y. Edges can join nodes of the opposite set but not nodes from the same set.

b) Alternating path:

D - 6 = E - 5 = F - 4 = B - 3

Change status:

D = 6 - E = 5 - F = 4 - B = 3

Improved matching:

A = 1

B = 3

C = 2

D = 6

E = 5

F = 4

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

2.

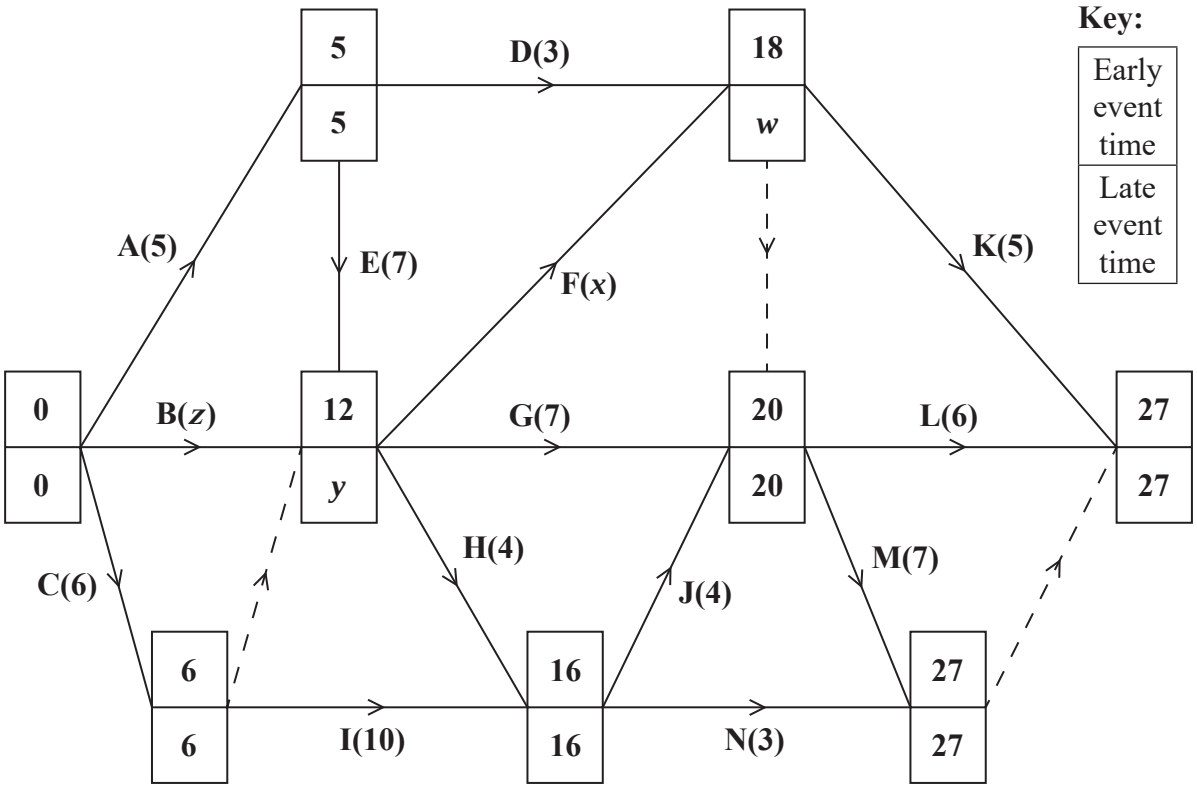


Figure 3

The network in Figure 3 shows the activities that need to be undertaken by a company to complete a project. Each activity is represented by an arc and the duration of the activity, in days, is shown in brackets. Each activity requires exactly one worker. The early event times and late event times are shown at each vertex.

Given that the total float on activity B is 2 days and the total float on activity F is also 2 days,

(a) find the values of w, x, y and z.

(4)

(b) Draw a cascade (Gantt) chart for this project on Grid 1 in the answer book.

(4)

(c) Use your cascade chart to determine the minimum number of workers needed to complete the project in the shortest possible time. You must make specific reference to time and activities. (You do not need to provide a schedule of the activities.)

(2)

(Total 10 marks)

Leave blank

2.

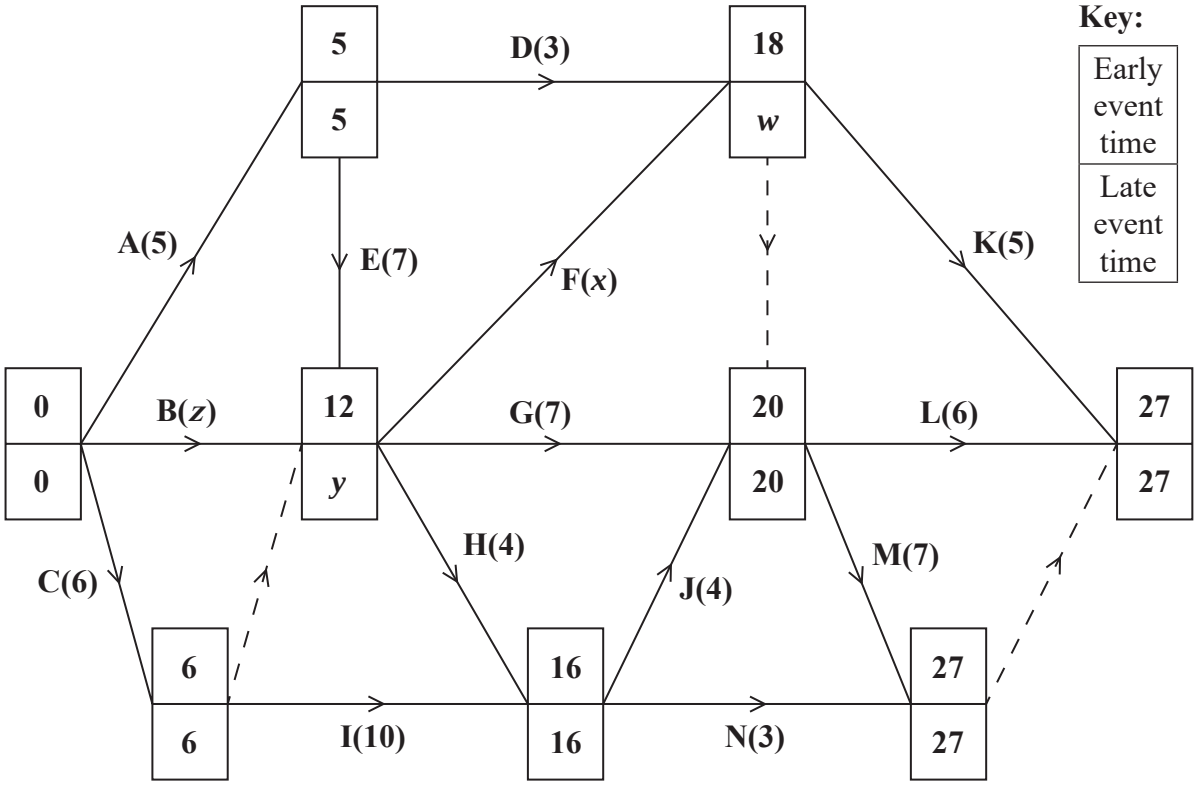


Figure 3

$w =$ 20 $x =$ 6 $y =$ 12 $z =$ 10

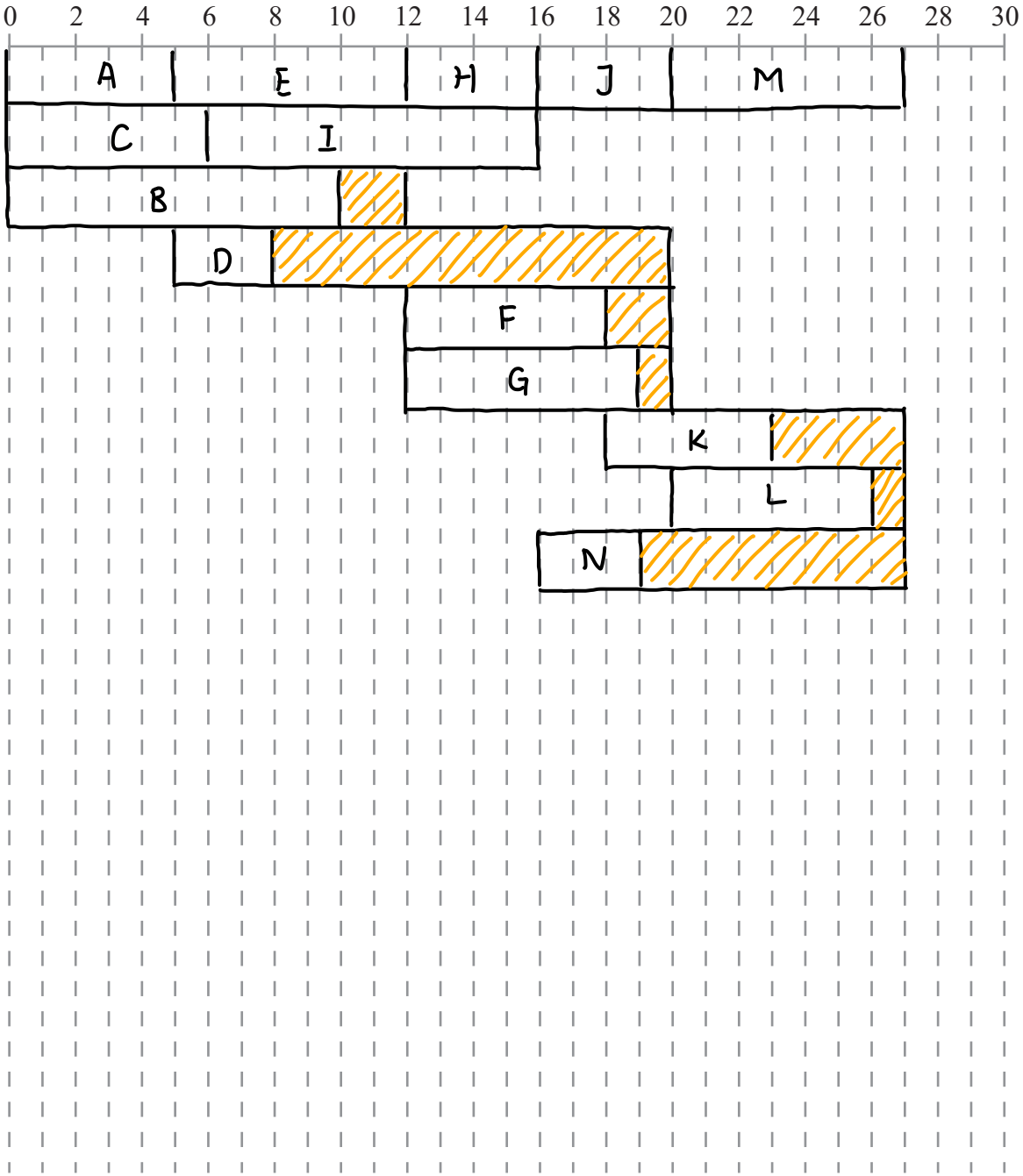
2a) $x = 18 - 12$
$= 6$
$y = 16 - 4$
$= 12$
$z = 12 - 2$
$= 10$

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 2 continued



Grid 1

c) 4 workers, working on activities H, I, F and G together from time 14 to 16

Q2

(Total 10 marks)

3.

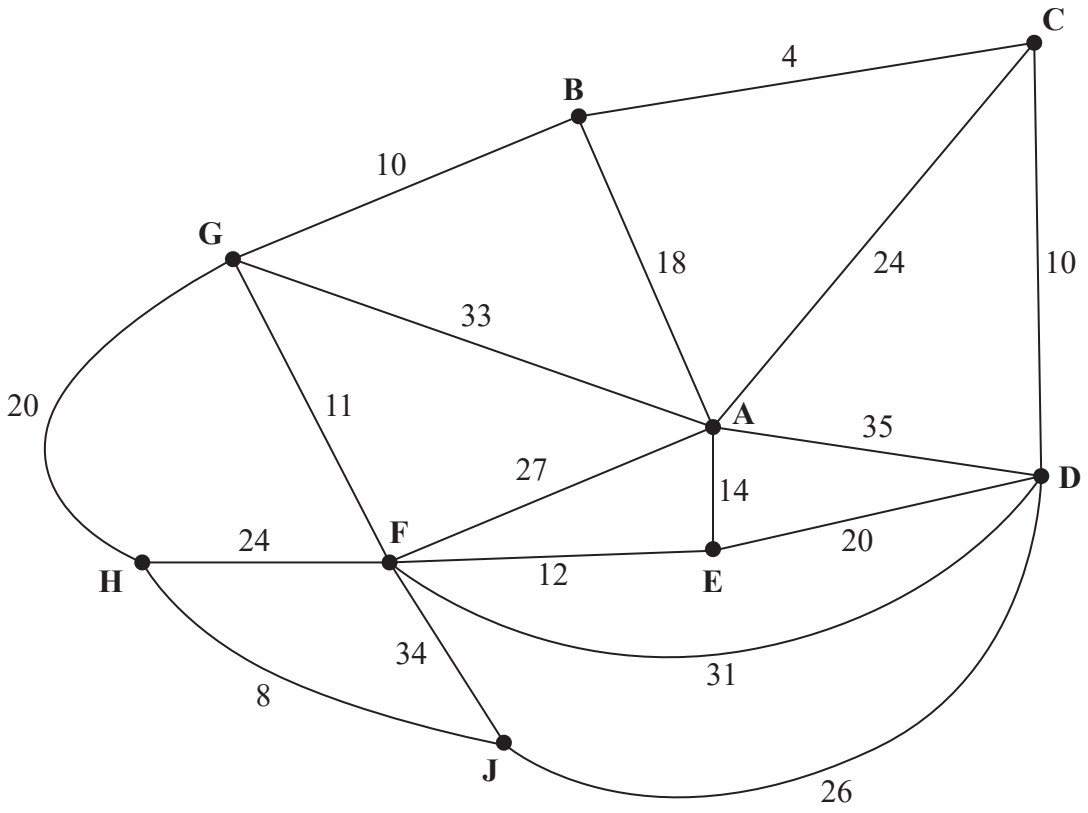


Figure 4

Figure 4 represents a network of train tracks. The number on each edge represents the length, in kilometres, of the corresponding track. Sally wishes to travel from A to J. She wishes to minimise the distance she travels.

(a) Use Dijkstra's algorithm to find the shortest path from A to J. State the path and its length.

(6)

(b) Find a route of minimal length that goes from D to H via A and state its length.

(2)

(c) Use Prim's algorithm, starting at E, to find a minimum spanning tree for the network. You must clearly state the order in which you select the edges of your tree.

(3)

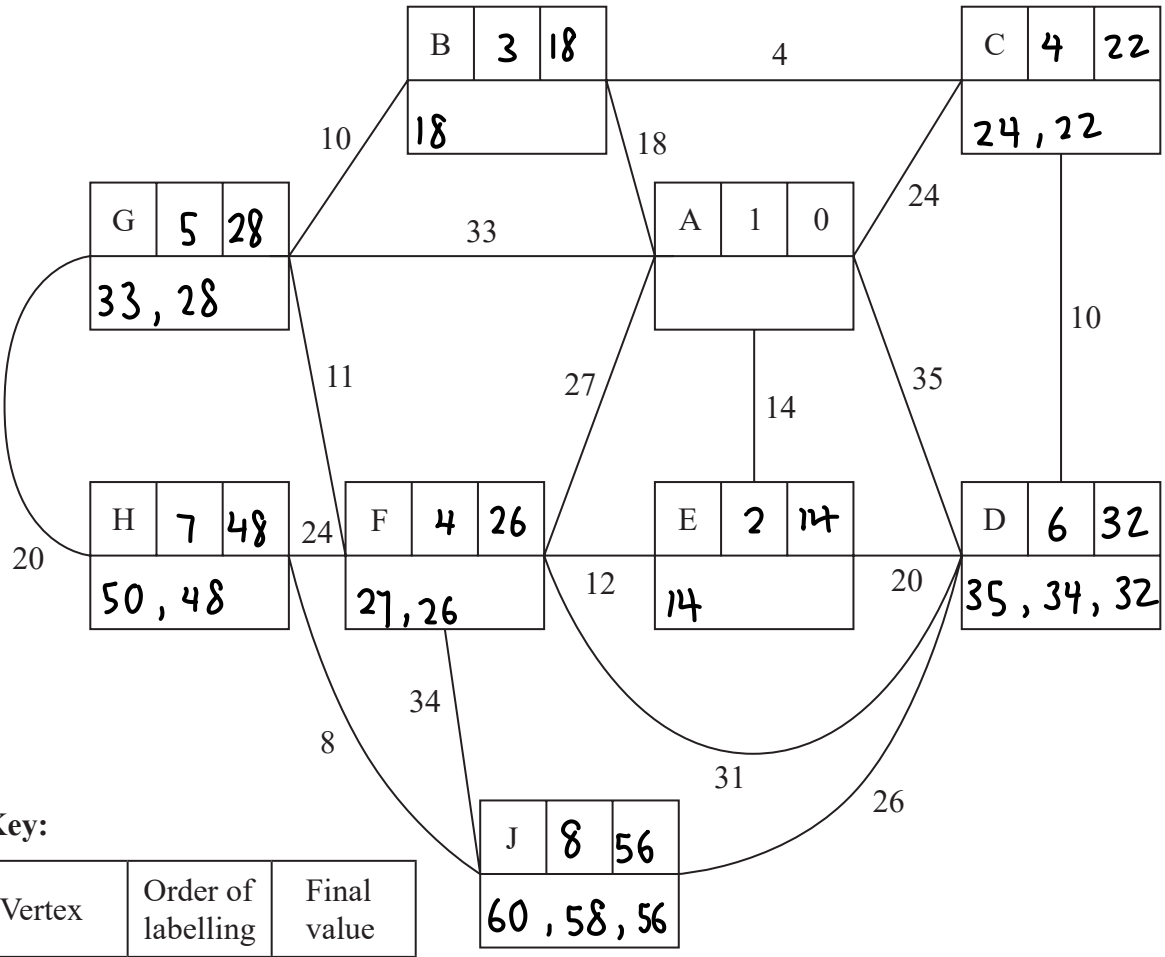
(d) Find the length, in kilometres, of your minimum spanning tree.

(1)

(Total 12 marks)

Leave blank

3.



Key:

Vertex	Order of labelling	Final value
Working values		

Shortest path: A - B - G - H - J

Length of shortest path: 56 km

b) D C B A B G H

length: 48 + 32 = 80 km

c) EF 12
FG 11
GB 10
BC 4
CD 10
EA 14
GH 20
HJ 8

d) length: 89 km

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

4.

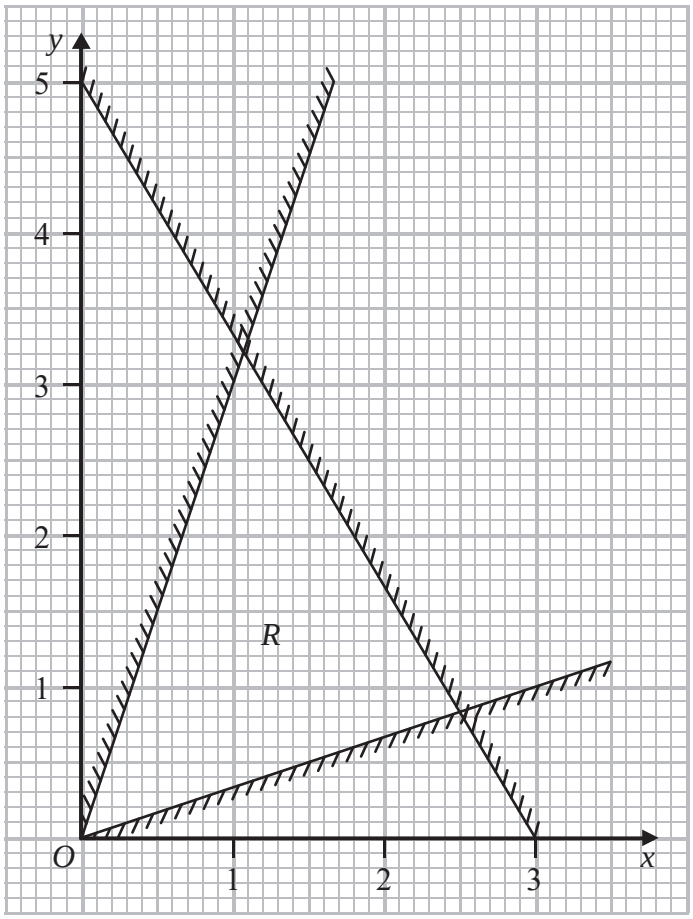


Figure 5

Figure 5 shows the constraints of a linear programming problem in x and y, where R is the feasible region.

(a) Determine the inequalities that define the feasible region.

(3)

(b) Find the exact coordinates of the vertices of the feasible region.

(3)

The objective is to maximise $P = 2x + 3y$.

(c) Use point testing at each vertex to find the optimal vertex, V, of the feasible region and state the corresponding value of P at V.

(3)

The objective is changed to maximise $Q = 2x + ky$, where k is a constant.

(d) Find the range of values of k for which the vertex identified in (c) is still optimal.

(2)

(Total 11 marks)

Leave blank

4.

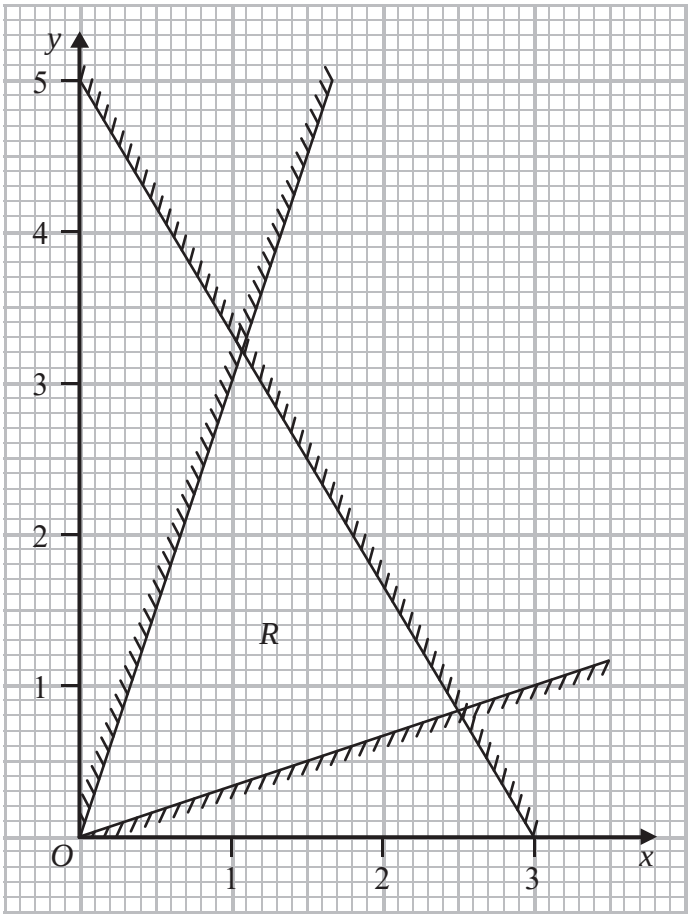


Figure 5

a) $y \leq 3x$

$5x + 3y \leq 15$

$y \geq \frac{1}{3}x \rightarrow 3y \geq x$

b) $y = 3x$

$5x + 3y = 15$

$5x + 3(3x) = 15$

$x = \frac{15}{14}$

$y = \frac{45}{14}$

$(\frac{15}{14}, \frac{45}{14})$

$y = 3x$

$y = \frac{1}{3}x$

$3x = \frac{1}{3}x$

$x = 0$

$y = 0$

$(0, 0)$

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 4 continued

$5x+3y=15$

$3y=x$

$6x=15$

$x=\frac{15}{6}$

$=\frac{5}{2}$

$y=\frac{1}{3}(\frac{5}{2})$

$=\frac{5}{6}$

$(\frac{5}{2},\frac{5}{6})$ $\therefore$ vertices : $(0,0),(\frac{15}{14},\frac{45}{14}),(\frac{5}{2},\frac{5}{6})$

c) $(0,0)$ $P=0$

$(\frac{15}{14},\frac{45}{14})$ $P=2(\frac{15}{14})+3(\frac{45}{14})$

$=\frac{165}{14}$

$(\frac{5}{2},\frac{5}{6})$ $P=2(\frac{5}{2})+3(\frac{5}{6})$

$=\frac{45}{6}$

$=\frac{15}{2}$

$\therefore$ optimal vertex : $(\frac{15}{14},\frac{45}{14})$

d) $2(\frac{15}{14})+k(\frac{45}{14}) \geq 2(\frac{5}{2})+k(\frac{5}{6})$

$\frac{30}{14}+\frac{45}{14}k \geq 5+\frac{5}{6}k$

$\frac{50}{21}k \geq \frac{20}{7}$

$k \geq \frac{6}{5}$

5.

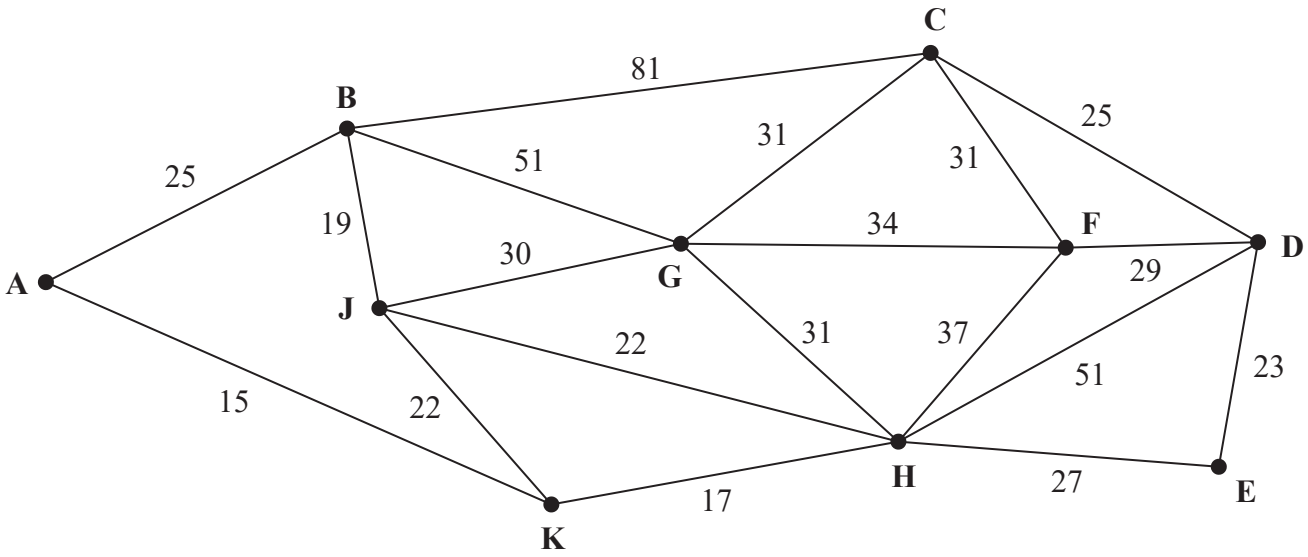


Figure 6

[*The total weight of the network is 601*]

Figure 6 represents a network of footpaths in a park. The number on each arc is the length, in metres, of the corresponding footpath. An inspection route of minimum length that traverses each footpath at least once needs to be found.

(a) Write down the nodes at which the route will start and finish.

(1)

It is now decided to start the inspection route at B and finish the inspection route at D. A route of minimum length that traverses each footpath at least once needs to be found.

(b) By considering the pairings of all relevant nodes find the arcs that will need to be traversed twice. You must make your method and working clear.

(4)

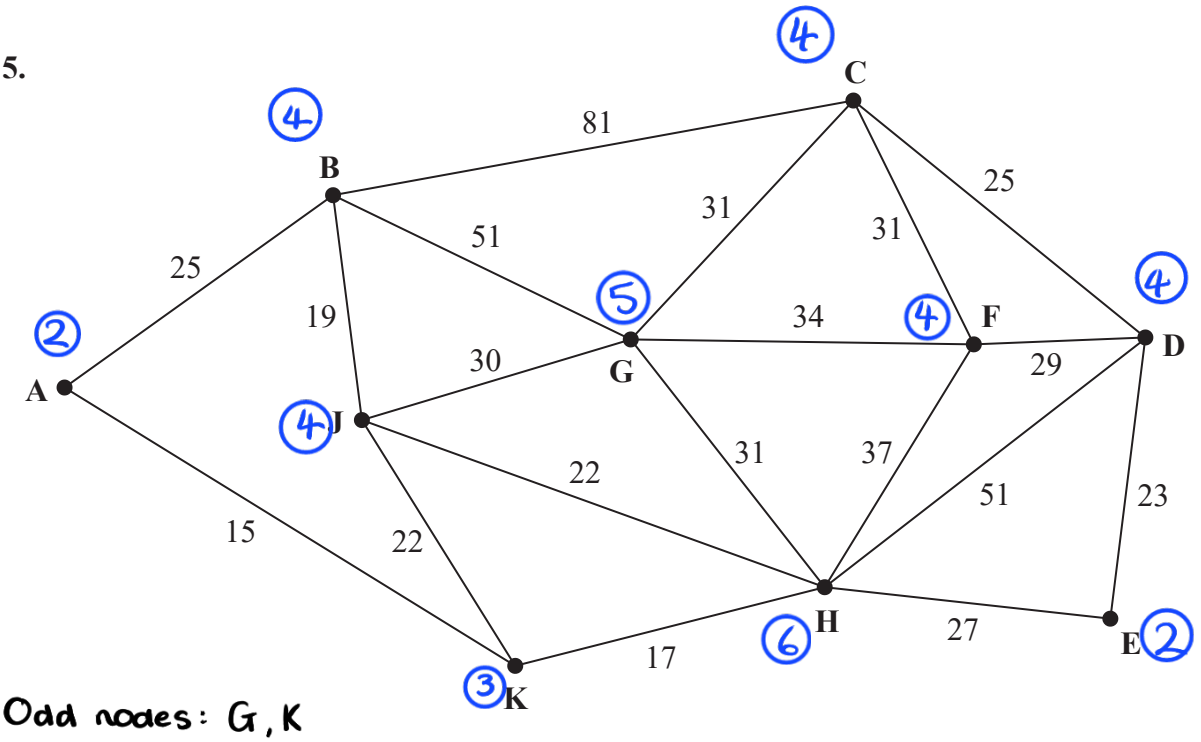
(c) Write down a possible shortest inspection route, giving its length.

(2)

(Total 7 marks)

Leave blank

5.



Odd nodes: G, K

Figure 6

[*The total weight of the network is 601*]

a) start at G and finish at K or start at K and finish at G

b) $\overset{(JHE)}{BD} + \overset{(H)}{GK} = (19+22+27+23)+(31+17) = 139$

$\overset{(J)}{BG} + \overset{(HE)}{DK} = (19+30)+(17+27+23) = 116$

$\overset{(A)}{BK} + \overset{(C)}{DG} = (25+15)+(31+25) = 96$ least weight

$\therefore$ BA, AK, GC and DC are traversed twice

c) BAKHEDCBGCDHJKABJGHFCGFD

length = 601 + 96

= 697 m

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

6.

30 11 21 53 50 39 16 4 60 43

The numbers in the list above represent the lengths, in cm, of some pieces of electrical wire. The wire is sold in one metre lengths.

(a) Use the first-fit bin packing algorithm to determine how these pieces could be cut from one metre lengths. You should ignore wastage due to cutting.

(3)

The list of numbers above is to be sorted into **ascending** order.

Starting at the left-hand end of the list, after three passes of the bubble sort, the list is

11 21 30 16 4 39 43 50 53 60

(b) (i) Write down the list that results at the end of the fourth pass.

(ii) Write down the number of comparisons and swaps performed during the fourth pass.

(3)

The **original** list of numbers is now to be sorted into **descending** order.

(c) Perform a quick sort to obtain the sorted list. You should show the result of each pass and identify your pivots clearly.

(4)

(d) Use the first-fit decreasing bin packing algorithm to determine how these pieces could be cut from one metre lengths. You should ignore wastage due to cutting.

(3)

(Total 13 marks)

11 21 30 50 39 16 4 53 43 60

6.

30 11 21 53 50 39 16 4 60 43

a) Bin 1: 30 11 21 16 4
Bin 2: 53 39
Bin 3: 50, 43
Bin 4: 60

bi) 11 21 16 4 30 39 43 50 53 60

ii) Comparisons: 6
Swaps: 2

c) 30 11 21 53 50 (39) 16 4 60 43
53 50 (60) 43 [39] 30 11 (21) 16 4
[60] 53 (50) 43 [39] 30 [21] 11 (16) 4
[60] 53 [50] 43 [39] 30 [21] [16] 11 (4)
[60] 53 [50] 43 [39] 30 [21] [16] 11 [4]
Sort complete

d) Bin 1: 60 39
Bin 2: 53 43 4
Bin 3: 50 30 16
Bin 4: 21 11

Leave blank

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

7. Emily is planning to sell three types of milkshake, strawberry, vanilla and chocolate.

Emily has completed some market research and has used this to form the following constraints on the number of milkshakes that she will sell each week.

- She will sell fewer than 200 non-vanilla milkshakes in total.
- She will sell at most 2.5 times as many strawberry milkshakes as vanilla milkshakes.
- At most, 75% of the milkshakes that she will sell will be vanilla.

The profit on each strawberry milkshake sold is £0.75, the profit on each vanilla milkshake sold is £1.20 and the profit on each chocolate milkshake sold is £1.45

Emily wants to maximise her profit.

Let x represent the number of strawberry milkshakes sold, let y represent the number of vanilla milkshakes sold and let z represent the number of chocolate milkshakes sold.

(a) Formulate this as a linear programming problem, stating the objective and listing the constraints as simplified inequalities with integer coefficients.

(6)

In week 1, Emily sells 100 strawberry milkshakes and 25 chocolate milkshakes.

(b) Calculate the maximum possible profit and minimum possible profit, in pounds, for the sale of all milkshakes in week 1. You must show your working.

(3)

(Total 9 marks)

Leave blank

7.

Maximise $P = 0.75x + 1.2y + 1.45z$

Subject to $x + z < 200$

$2.5y \geq x \rightarrow 5y \geq 2x$

$y \leq \frac{3}{4}(x+y+z) \rightarrow y \leq 3x + 3z$

$x \geq 0$

$y \geq 0$

$z \geq 0$

b) $x = 100$, $z = 25$

$y = 3x + 3z$

$= 3(100 + 25)$

$= 375$

$5y = 2x$

$y = \frac{2}{5}(100)$

$= 40$

$40 \leq y \leq 375$

Max profit : $0.75(100) + 1.2(375) + 1.45(25) = £561.25$

Min profit : $0.75(100) + 1.2(40) + 1.45(25) = £159.25$

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

8.

Activity	Immediately preceding activities
A	–
B	–
C	A
D	A
E	B
F	B
G	B, D
H	B, D
I	E, H
J	C, G, I
K	E, F, H

(a) Draw the activity network described in the precedence table, using activity on arc. Your activity network must contain only the minimum number of dummies.

(5)

The earliest start time for activity G is 12 and the latest finish time for activity E is 25

Given that the duration of activity H is 10

(b) find the total float of activity H.

(1)

(c) State whether the path ADHIJ is a critical path. Justify your answer.

(2)

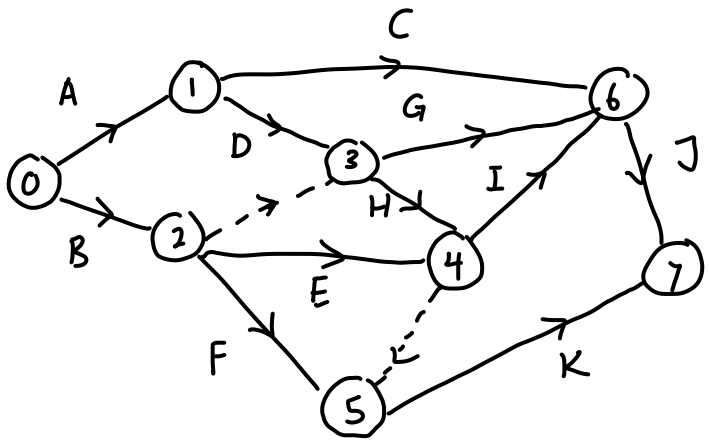
(Total 8 marks)

TOTAL FOR PAPER: 75 MARKS

END

Leave blank

8.



b) $25 - 10 - 12 = 3$

c) No. Activity H is not critical as it has a float of 3.

Q8

(Total 8 marks)

TOTAL FOR PAPER: 75 MARKS

END

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA