GradeMax


S1S D1

**Write your answers in the D1 answer book for this paper.**

**1.**

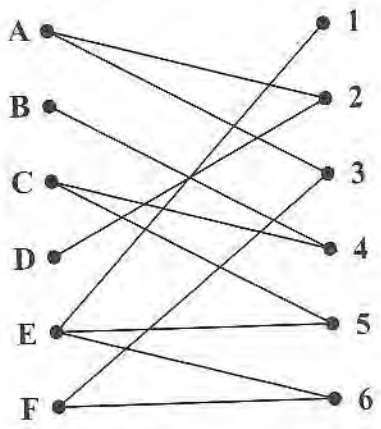



**Figure 1**

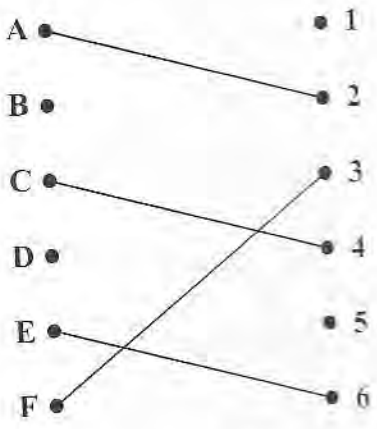



**Figure 2**

A delivery firm has six vans, A, B, C, D, E and F, available for six deliveries, 1, 2, 3, 4, 5 and 6. Each van must be assigned to just one delivery.

The bipartite graph shown in Figure 1 shows the possible matchings and Figure 2 shows an initial matching.

A complete matching is required, starting from the given initial matching.

(a) Explain why it is necessary to use the maximum matching algorithm twice. **(1)**

There are three possible alternating paths that start at either D or B. One of these is

$$D - 2 = A - 3 = F - 6 = E - 5$$

(b) Find the other two alternating paths. **(2)**

(c) List the improved matching generated by using the alternating path

$$D - 2 = A - 3 = F - 6 = E - 5$$

**(1)**

(d) Starting from the improved matching found in (c), use the maximum matching algorithm to find a complete matching. You should list the alternating path that you use and your complete matching. **(2)**

**(Total 6 marks)**

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D1 SIS

1.

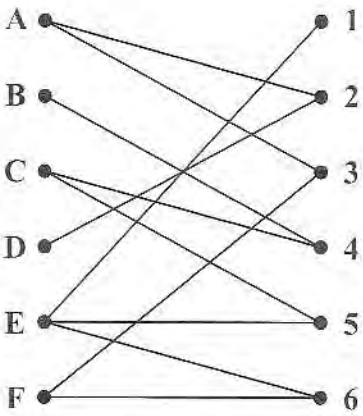



Figure 1

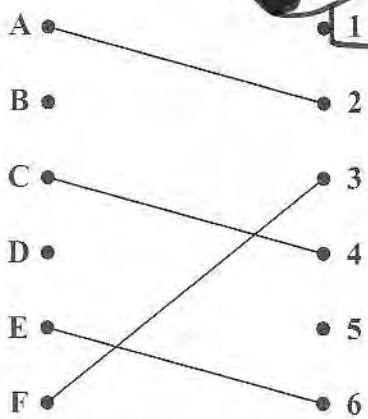



Figure 2

a) each iteration of the algorithm matches one extra pair (if possible). The initial solution has 2 missing pairs ∴ the algorithm will need to be done twice.

b) $D-2 = A-3 = F-6 = E-1$

cs $D=2 - A=3 - F=6 - E=1$

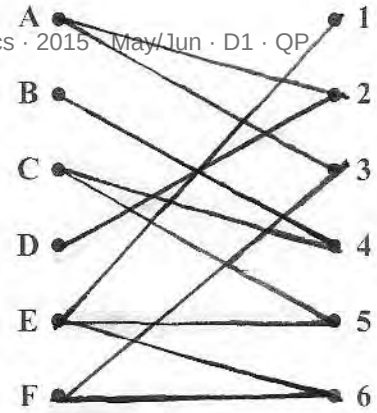
$B-4 = C-5$

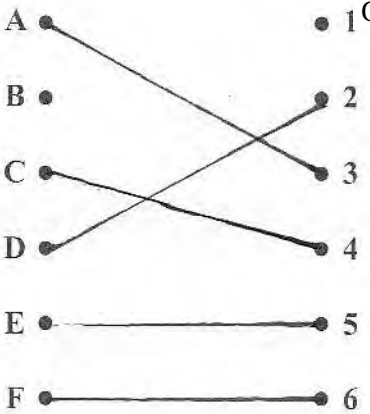
cs $B=4 - C=5$

c) $A=3$ $B=$ unmatched $C=4$ $D=2$ $E=5$ $f=6$

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$B-4=C-5=E-1$

CS

$B=4 \quad C=5 \quad E=1$

Complete Matching

$A=3$
$B=4$
$C=5$
$D=2$
$E=1$
$F=6$

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18 29 48 9 42 31 37 24 27 41

The numbers above are Alan's batting scores for the first 10 cricket matches of the season.

(a) Use a quick sort to sort this list of numbers into ascending order. You must make your pivots clear.

**(4)**

Alan's batting scores for the final 10 cricket matches of the same season were

72 53 89 91 68 67 90 77 83 75

(b) Carry out a bubble sort on this second list of numbers to produce a list of these scores in ascending order. You need only give the state of the list after each pass.

**(4)**

Alan's combined batting scores for the entire season were

9 18 24 27 29 31 37 41 42 48 53 67 68 72 75 77 83 89 90 91

(c) Use the binary search algorithm to locate 68 in the combined list of 20 scores. You must make your method clear.

**(3)**

**(Total 11 marks)**

GradeMax

QS

| | | | | | | | | | | Pivots |
|---|---|---|---|---|---|---|---|---|---|---|
| 18 | 29 | 48 | 9 | 42 | 31 | 37 | 24 | 27 | 41 | 31 |
| 18 | 29 | 9 | 24 | 27 | 31 | 48 | 42 | 37 | 41 | 9, 37 |
| 9 | 18 | 29 | 24 | 27 | 31 | 37 | 48 | 42 | 41 | 24, 42 |
| 9 | 18 | 24 | 29 | 27 | 31 | 37 | 41 | 42 | 48 | 18, 27, 41, 42, 48 |
| 9 | 18 | 24 | 27 | 29 | 31 | 37 | 41 | 42 | 48 | 29 |
| 9 | 18 | 24 | 27 | 29 | 31 | 37 | 41 | 42 | 48 | |

b)

BS

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 72 | 53 | 89 | 91 | 68 | 67 | 90 | 77 | 83 | 75 | |
| 53 | 72 | 89 | 68 | 67 | 90 | 77 | 83 | 75 | 91 | (1) |
| 53 | 72 | 68 | 67 | 89 | 77 | 83 | 75 | 90 | 91 | (2) |
| 53 | 68 | 67 | 72 | 77 | 83 | 75 | 89 | 90 | 91 | (3) |
| 53 | 67 | 68 | 72 | 77 | 75 | 83 | 89 | 90 | 91 | (4) |
| 53 | 67 | 68 | 72 | 75 | 77 | 83 | 89 | 90 | 91 | (5) |
| 53 | 67 | 68 | 72 | 75 | 77 | 83 | 89 | 90 | 91 | (6) |

c) $\frac{1+20}{2} = 10.5 \rightarrow 11^{th} = 53$ delete 1-11

$\frac{12+20}{2} = 16^{th} = 77$ delete 16-20

$\frac{12+15}{2} = 13.5 \rightarrow 14^{th} = 72$ delete 14-15

$\frac{12+13}{2} = 12.5 = 13^{th} = 68$

$\therefore$ 68 is 13th in the list.

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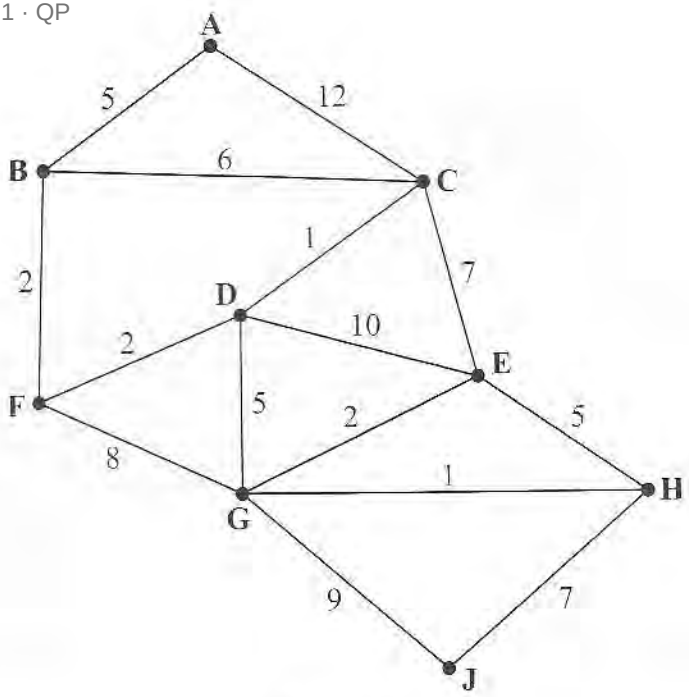



**Figure 3**

Figure 3 represents a network of roads. The number on each arc is the length, in km, of the corresponding road.

(a) Use Dijkstra's algorithm to find the shortest route from A to J. State the shortest route and its length.

**(6)**

(b) Explain how you determined the shortest route from your labelled diagram.

**(2)**

(c) Find the shortest route from A to J via E and state its length.

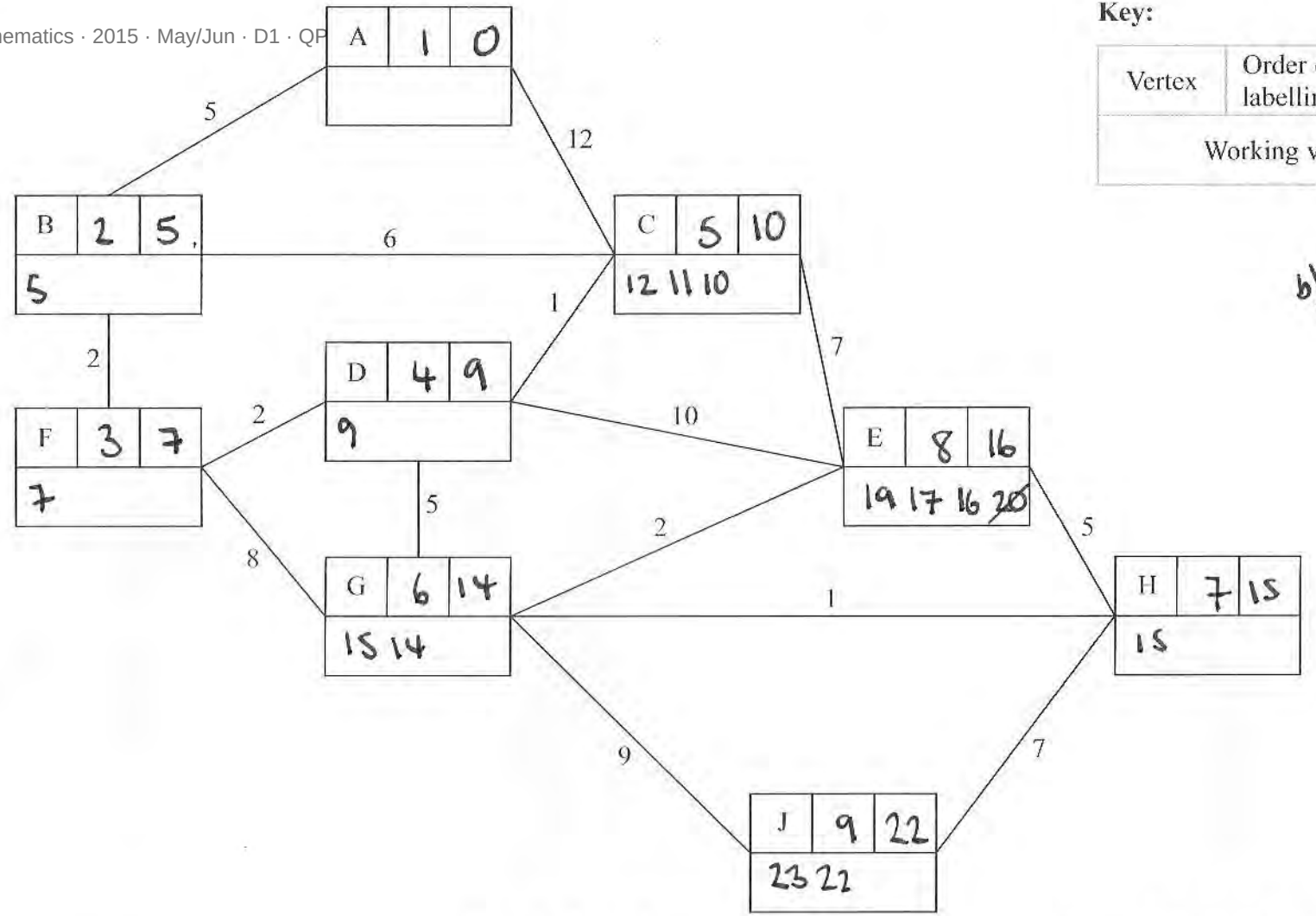
**(2)**

**(Total 10 marks)**

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**Key:**

| Vertex | Order of labelling | Final value |
|---|---|---|
| Working values | | |




b)
22-7=15 HJ
15-1=14 GH
14-5=9 DG
9-2=7 FD
7-2=5 BF
5-5=0 AB

Shortest route: ABFDGHJ

c) ABFDGEGHJ (26)

Length of shortest route: 22

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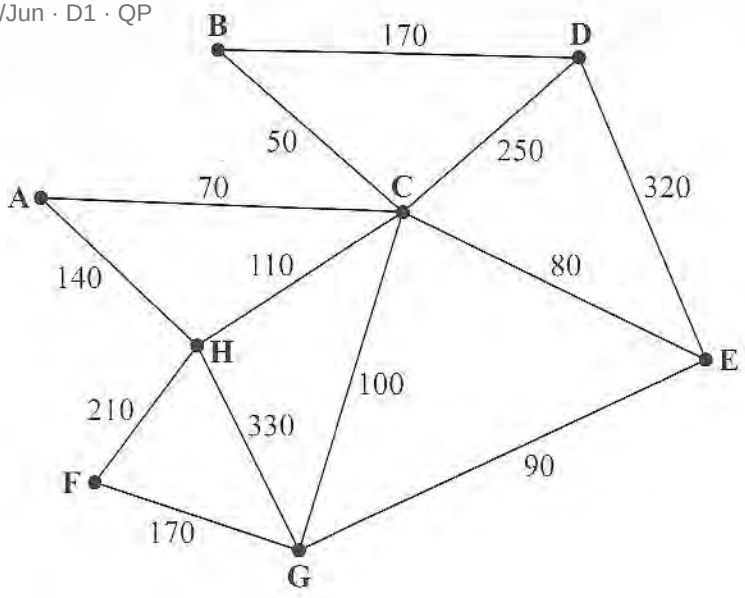



**Figure 4**

[*The total weight of the network is 2090*]

(a) Explain why a network cannot have an odd number of vertices of odd valency.

**(2)**

Figure 4 represents a network of 13 roads in a village. The number on each arc is the length, in metres, of the corresponding road. A route of minimum length that traverses each road at least once needs to be found. The route may start at any vertex and finish at any vertex.

(b) Write down the vertices at which the route will start and finish.

**(1)**

A new road, AB, of length 130m is built. A route of minimum length that traverses each road, including AB, needs to be found. The route must start and finish at A.

(c) Use the route inspection algorithm to find the roads that will need to be traversed twice. You must make your method and working clear.

**(4)**

(d) Calculate the length of a possible shortest inspection route.

**(2)**

It is now decided to start and finish the inspection route at two distinct vertices. A route of minimum length that traverses each road, including AB, needs to be found. The route must start at A.

(e) State the finishing point so that the length of the route is minimised. Calculate how much shorter the length of this route is compared to the length of the route in (d). You must make your method and calculations clear.

**(3)**

**(Total 12 marks)**

GradeMax

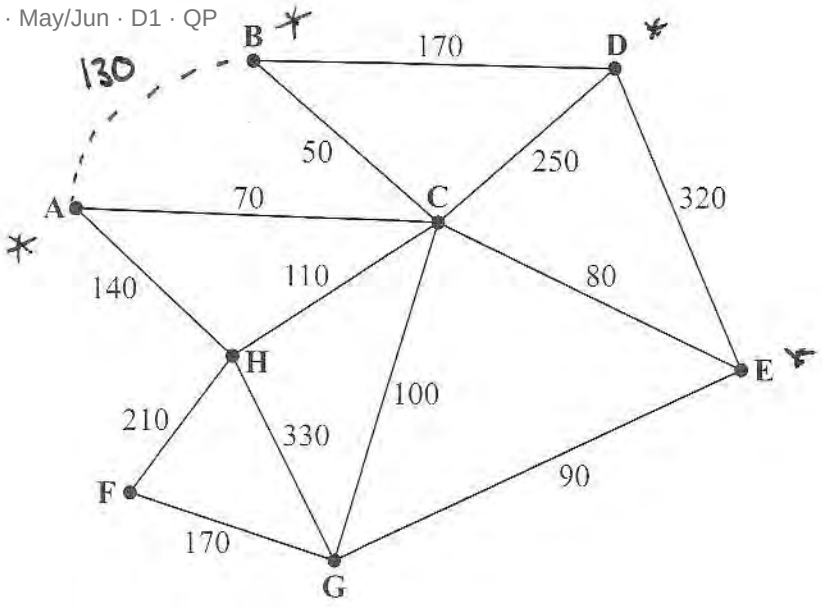



**Figure 4**

[*The total weight of the network is 2090*]

a) Every arc has two end points, so will contribute 2 to the total valency.
∴ total valency must be even
∴ there must be an even number of odd pairs to make the total even
∴ there cannot be an odd number of vertices with an odd valency.

b) D, E

c) new weight = 2090 + 130 = 2220

| | | | | |
|---|---|---|---|---|
| AB, BC | 120 | DE, BC | 300 | ⇒ 420 |
| AD, CB | 290 | BE, C | 130 | ⇒ 420 |
| AE, C | 150 | BD | 170 | ⇒ 320 * |

repeat AC, CE, BD

d) + 320
= 2540
2

e) repeat BC, CE +130 = 2350 shorter by 190 end at D

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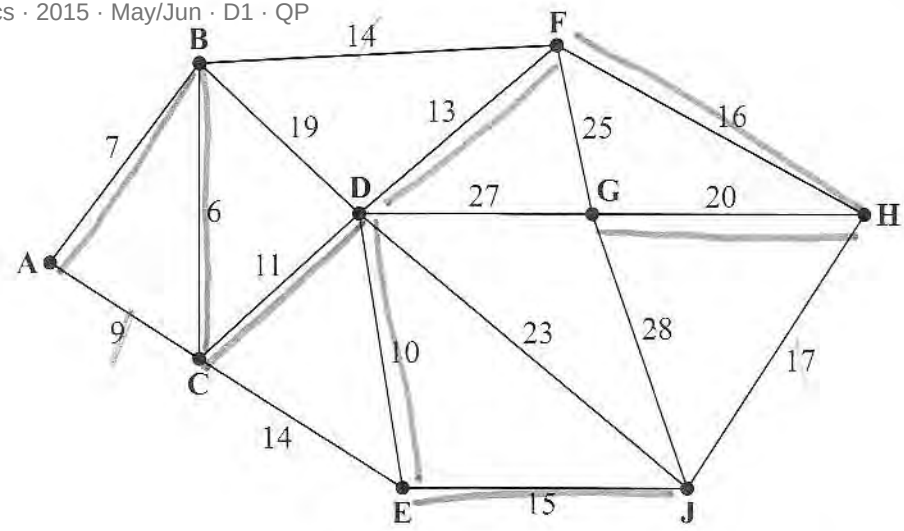




**Figure 5**

a) BC ; AB ; AC ✗ ; DE ; CD ; DF ; BF ✗ / CE ✗ ; EJ ; FH ; HJ ✗ ; GH

reject all other.

b) GH ; HF ; FD ; DE ; DC ; BC ; BA ; EJ
(20) (16) (13) (10) (11) (6) (7) (15)

c) 98

d) i) $\frac{m}{2}$ ii) $n-1$ iii) arcs in V > arcs in T

$\frac{m}{2} > n-1$

$m > 2n-2$

2

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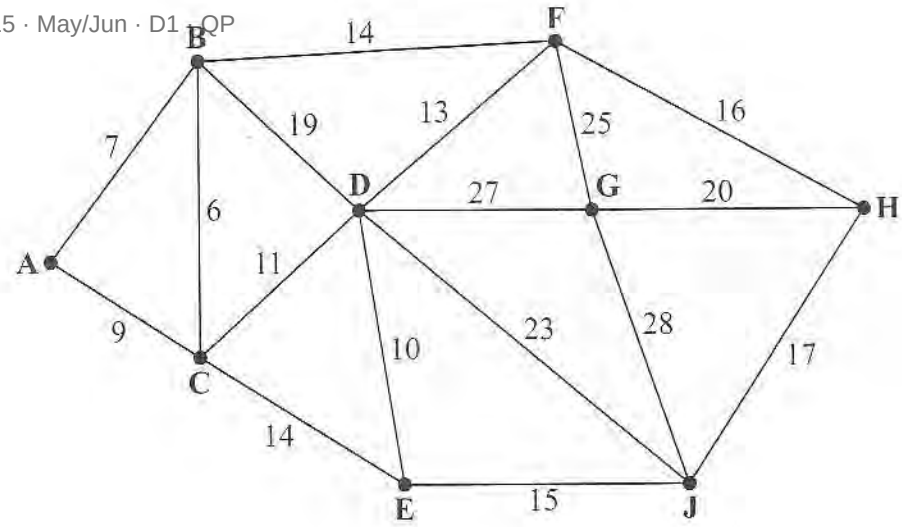



**Figure 5**

The numbers on the 17 arcs in the network shown in Figure 5 represent the distances, in km, between nine nodes, A, B, C, D, E, F, G, H and J.

(a) Use Kruskal's algorithm to find a minimum spanning tree for the network. You should list the arcs in the order in which you consider them. In each case, state whether you are adding the arc to your minimum spanning tree.

**(3)**

(b) Starting at G, use Prim's algorithm to find a minimum spanning tree. You must clearly state the order in which you select the arcs of your tree.

**(3)**

(c) Find the weight of the minimum spanning tree.

**(1)**

A connected graph V has $n$ nodes. The sum of the degrees of all the nodes in V is $m$. The graph T is a minimum spanning tree of V.

(d) (i) Write down, in terms of $m$, the number of arcs in V.

(ii) Write down, in terms of $n$, the number of arcs in T.

(iii) Hence, write down an inequality, in terms of $m$ and $n$, comparing the number of arcs in T and V.

**(3)**

**(Total 10 marks)**

GradeMax


**6.** A linear programming problem in $x$ and $y$ is described as follows.

Minimise $C = 2x + 3y$

subject to

$$x + y \geqslant 8$$
$$x < 8$$
$$4y \geqslant x$$
$$3y \leqslant 9 + 2x$$

(a) Add lines and shading to Diagram 1 in the answer book to represent these constraints.

**(4)**

(b) Hence determine the feasible region and label it R.

**(1)**

(c) Use the objective line (ruler) method to find the exact coordinates of the optimal vertex, V, of the feasible region. You must draw and label your objective line clearly.

**(3)**

(d) Calculate the corresponding value of $C$ at V.

**(1)**

The objective is now to maximise $2x + 3y$, where $x$ and $y$ are integers.

(e) Write down the optimal values of $x$ and $y$ and the corresponding maximum value of $2x + 3y$.

**(2)**

A further constraint, $y \leqslant kx$, where $k$ is a positive constant, is added to the linear programming problem.

(f) Determine the least value of $k$ for which this additional constraint does not affect the feasible region.

**(2)**

**(Total 13 marks)**

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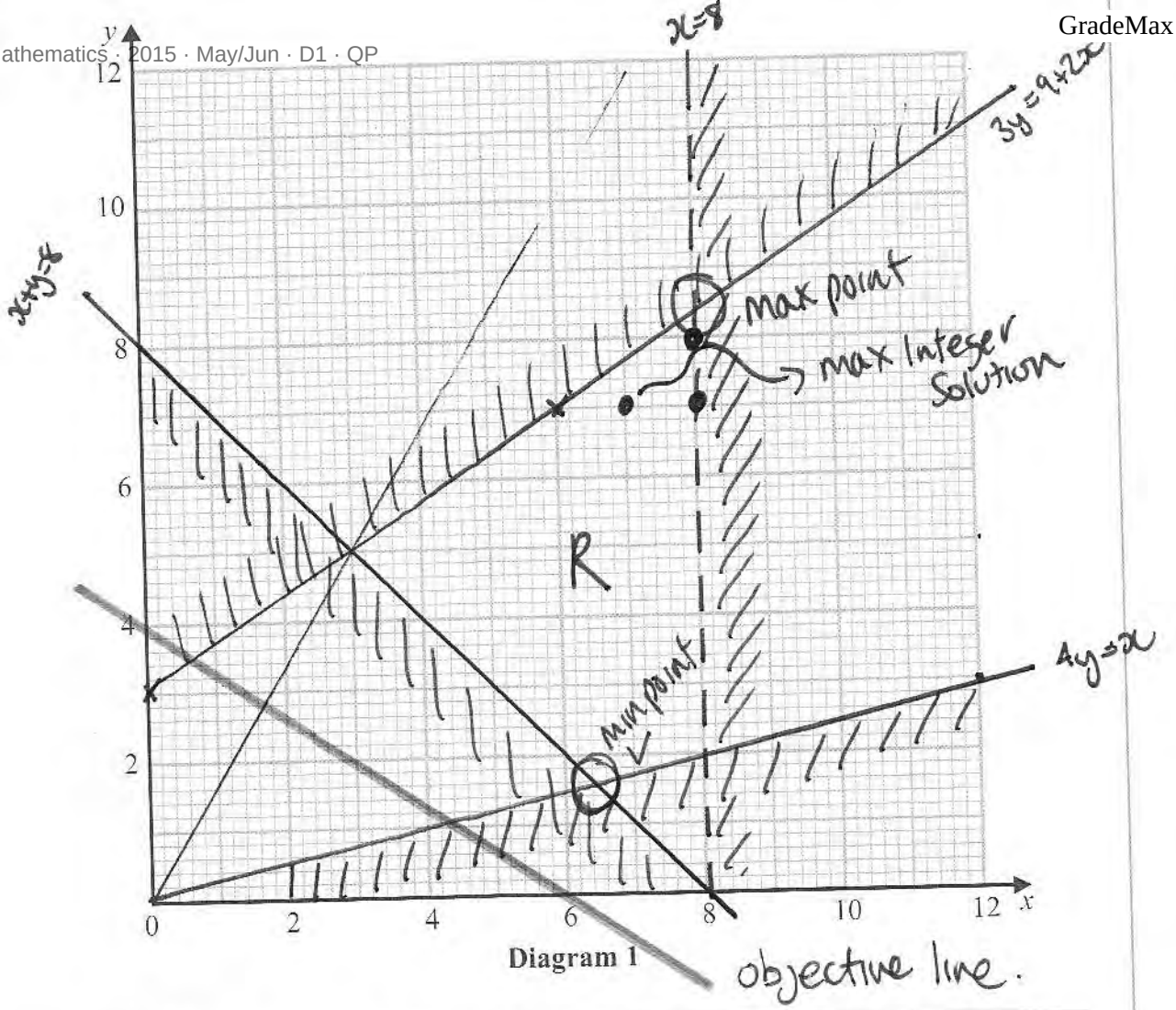



Diagram 1

a) $3y = 9+2x$ $\quad x=6 \quad 9+2x = 21 = 3y \therefore y=7$
$\quad x=0 \quad 9+2x = 9 = 3y \therefore y=3$

c) $2x+3y=12 \Rightarrow x=0 \quad y=4$
$\quad y=0 \quad x=6$

$4y=x$ intersects $x+y=8 \Rightarrow 5y=8 \therefore y=1.6$
$\quad x=6.4$

$V(6.4, 1.6)$

d) $C=2x+3y = 2(6.4)+3(1.6) = 17.6$

e) $x=8 \quad 3y=9+2(8)=25 \quad y=\frac{25}{3} \quad (8, \frac{25}{3})$ exact.

Integer solution $\Rightarrow$
$(7,7)$

f) Cannot be lower than $(3,5) \therefore k=\frac{5}{3}$

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| Activity | Time taken (days) | Immediately preceding activities |
|---|---|---|
| A | 5 | - |
| B | 7 | - |
| C | 8 | - |
| D | 5 | A |
| E | 7 | A |
| F | 10 | B, C |
| G | 4 | B, C |
| H | 9 | C |
| I | 8 | G, H |
| J | 12 | G, H |
| K | 7 | D |
| L | 10 | E, F, I, J |

The table shows the activities required for the completion of a building project. For each activity the table shows the time taken, in days, and the immediately preceding activities. Each activity requires one worker. The project is to be completed in the shortest possible time.

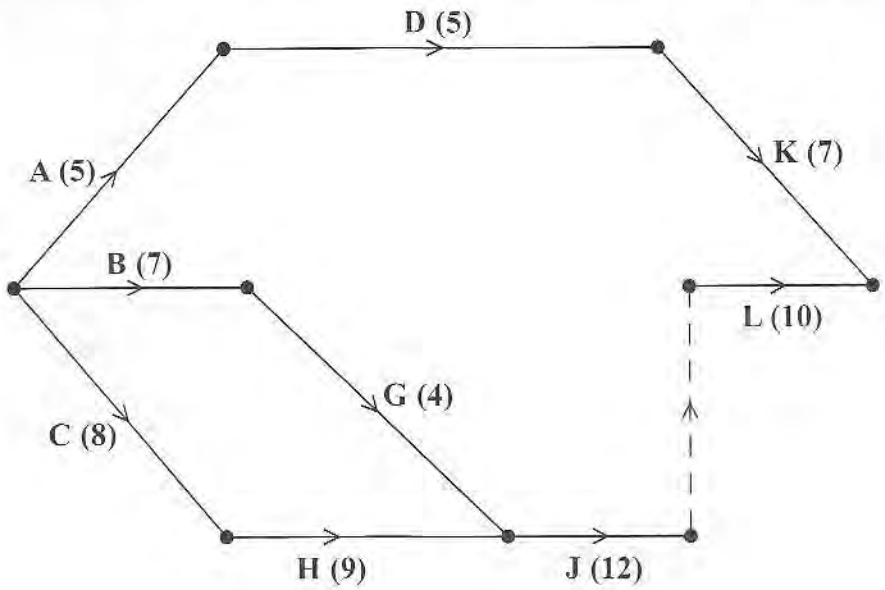



**Figure 6**

Figure 6 shows a partially completed activity network used to model the project. The activities are represented by the arcs and the numbers in brackets on the arcs are the times taken, in days, to complete each activity.

(a) Add activities, E, F and I, and exactly one dummy to Diagram 1 in the answer book.

**(3)**

(b) Complete Diagram 1 in the answer book to show the early event times and late event times.

**(4)**

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(c) Calculate a lower bound for the number of workers needed to complete the project in the shortest possible time. You must show your working.

**(2)**

(d) Schedule the activities, using the minimum number of workers, so that the project is completed in the minimum time.

**(4)**

**(Total 13 marks)**

**TOTAL FOR PAPER: 75 MARKS**

**END**

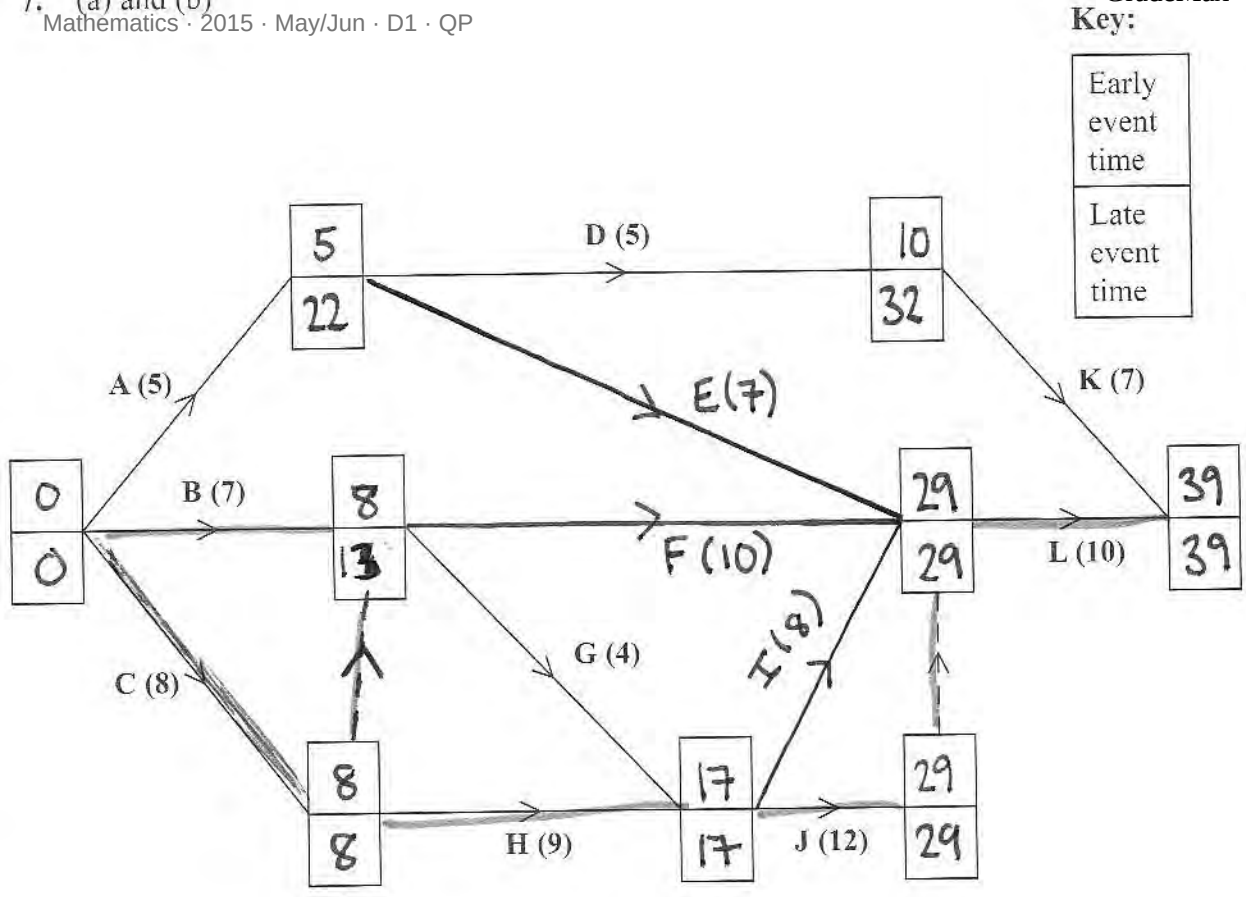



Diagram 1

Critical Path CHJL

c) $LB = \frac{92}{39} = 2.36 \Rightarrow$ 3 workers

(d)

