MODEL ANSWERS
GradeMax

January 2014 (IAL)

Write your name here

Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number | Candidate Number

Decision Mathematics D1

Advanced/Advanced Subsidiary

Wednesday 22 January 2014 – Morning
Time: 1 hour 30 minutes

Paper Reference
WDM01/01

Answer Book
Do not return the question paper with the answer book.

Total Marks

Turn over ▶

P43013A

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5/5/5/5/5/5/

P 4 3 0 1 3 A 0 1 1 6

PEARSON

Write your answers in the D1 answer book for this paper.

1.

11 17 10 14 8 13 6 4 15 7

(a) Use the bubble sort algorithm to perform **ONE** complete pass towards sorting these numbers into **ascending** order.

(2)

The **original** list is now to be sorted into **descending** order.

(b) Use a quick sort to obtain the sorted list, giving the state of the list after each complete pass. You must make your pivots clear.

(4)

The numbers are to be packed into bins of size 26

(c) Calculate a lower bound for the minimum number of bins required. You must show your working.

(2)

(Total 8 marks)

Leave blank

1. a) 11 10 14 8 13 6 4 15 7 17

b) 11 17 10 14 8 (13) 6 4 15 7
17 (14) 15 [13] 11 10 8 (6) 4 7
17 (15) [14] [13] 11 10 (8) 7 [6] 4
17 [15] [14] [13] 11 (10) [8] 7 [6] 4
17 [15] [14] [13] 11 [10] [8] 7 [6] 4
sort complete

c) $(11+17+10+14+8+13+6+4+15+7) \div 26 = 105 \div 26$
$= 4.04$
$\triangleq 5$ bins

2.

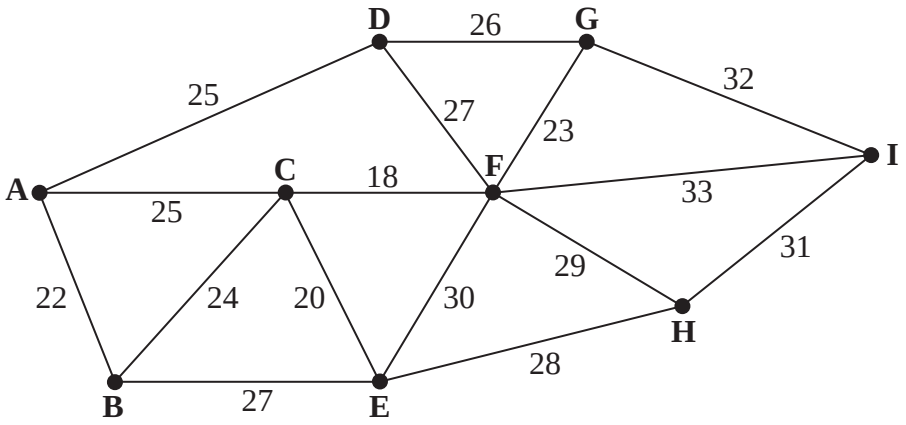


Figure 1

Figure 1 represents nine buildings, A, B, C, D, E, F, G, H and I, recently bought by Newberry Enterprises. The company wishes to connect the alarm systems between the buildings to form a single network. The number on each arc represents the cost, in pounds, of connecting the alarm systems between the buildings.

(a) Use **Prim's** algorithm, starting at A, to find the minimum spanning tree for this network. You must list the **arcs** that form your tree in the order that you select them.

(3)

(b) State the minimum cost of connecting the alarm systems in the nine buildings.

(1)

It is discovered that some alarm systems are already connected. There are connections along BC and EF, as shown in bold in Diagram 1 in the answer book. Since these already exist, it is decided to use these arcs as part of the spanning tree.

(c) (i) Use **Kruskal's** algorithm to find the minimum spanning tree that includes arcs BC and EF. You must list the arcs in the order that you consider them. In each case, state whether you are adding the arc to your spanning tree.

(ii) Explain why Kruskal's algorithm is a better choice than Prim's algorithm in this case.

(5)

Since arcs BC and EF already exist, there is no cost for these connections.

(d) State the new minimum cost of connecting the nine buildings.

(1)

(Total 10 marks)

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2.

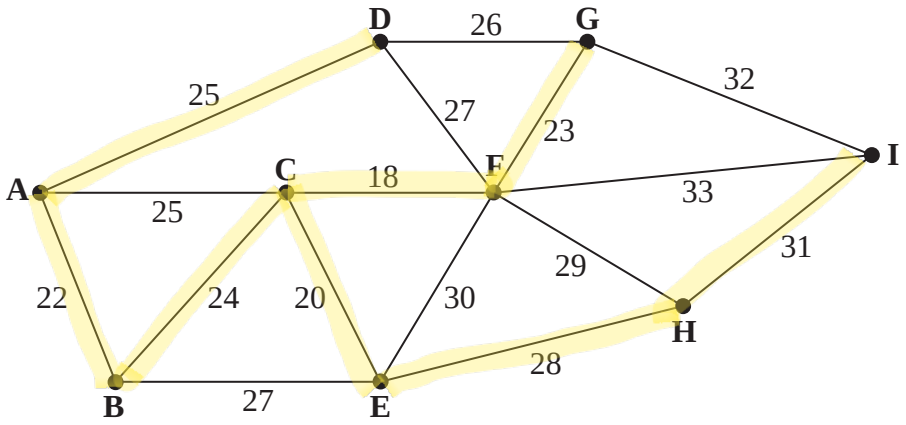


Figure 1

(a)

AB , BC , CF , CE , FG , AD , EH , HI

22 + 24 + 18 + 20 + 23 + 25 + 28 + 31 = £191

(b) Minimum cost: £ 191

Leave blank

Question 2 continued

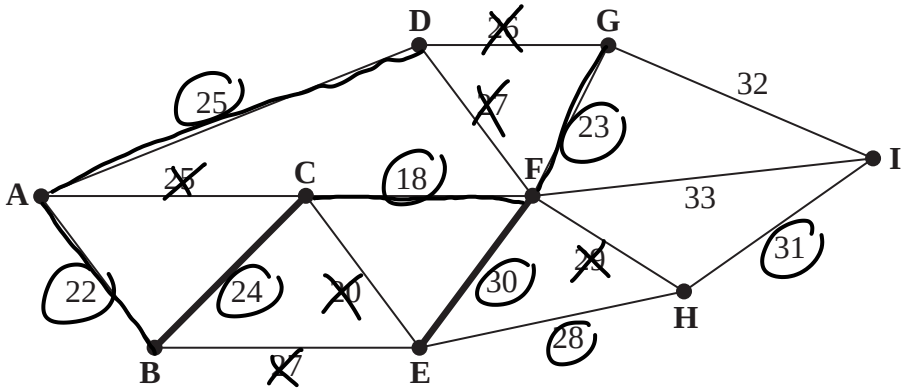


Diagram 1

(c) (i)

CF, reject CE, AB, FG, reject AC, AD, reject DG, reject DF, reject BE, EH, reject FH, HI

* Reject as they cause a cycle to be formed.

(c) (ii)

In Prim's algorithm, the tree grows in a connected fashion

d) 25+18+23+22+28+31=147

(d) New minimum cost: £ 147

Q2

(Total 10 marks)

3.

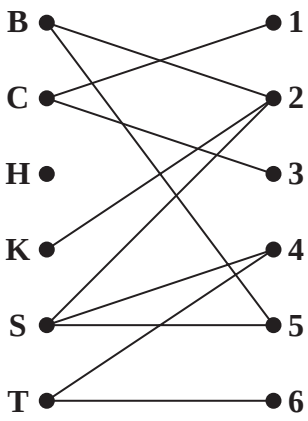


Figure 2

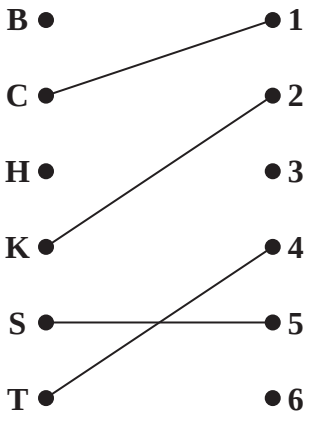


Figure 3

Figure 2 shows the possible allocations of six people, Beth (B), Charlie (C), Harry (H), Karam (K), Sam (S) and Theresa (T), to six tasks 1, 2, 3, 4, 5 and 6.

Figure 3 shows an initial matching.

(a) Define the term ‘matching’.

(2)

(b) Starting from the given initial matching, use the maximum matching algorithm to find an improved matching. You should list the alternating path that you use, and state the improved matching.

(3)

After training, a possible allocation for Harry is task 6, and an additional possible allocation for Karam is task 1.

(c) Starting from the matching found in (b), use the maximum matching algorithm to find a complete matching. You should list the alternating path that you use, and state your complete matching.

(3)

(Total 8 marks)

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3.

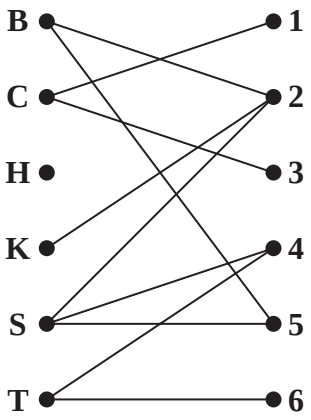


Figure 2

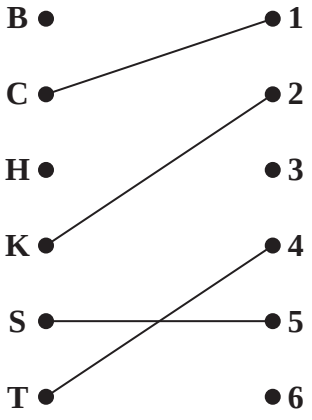


Figure 3

a) Some or all elements of set X is paired with elements of another set Y.

b) Alternating path:
B - 5 = S - 4 = T - 6
Change status:
B = 5 - S = 4 - T = 6
Improved matching:
B = 5
C = 1
H =
K = 2
S = 4
T = 6

c) Alternating path:
H - 6 = T - 4 = S - 2 = K - 1 = C - 3
Change status:
H = 6 - T = 4 - S = 2 - K = 1 - C = 3
Complete matching:
B = 5
C = 3
H = 6
K = 1
S = 2
T = 4

4.

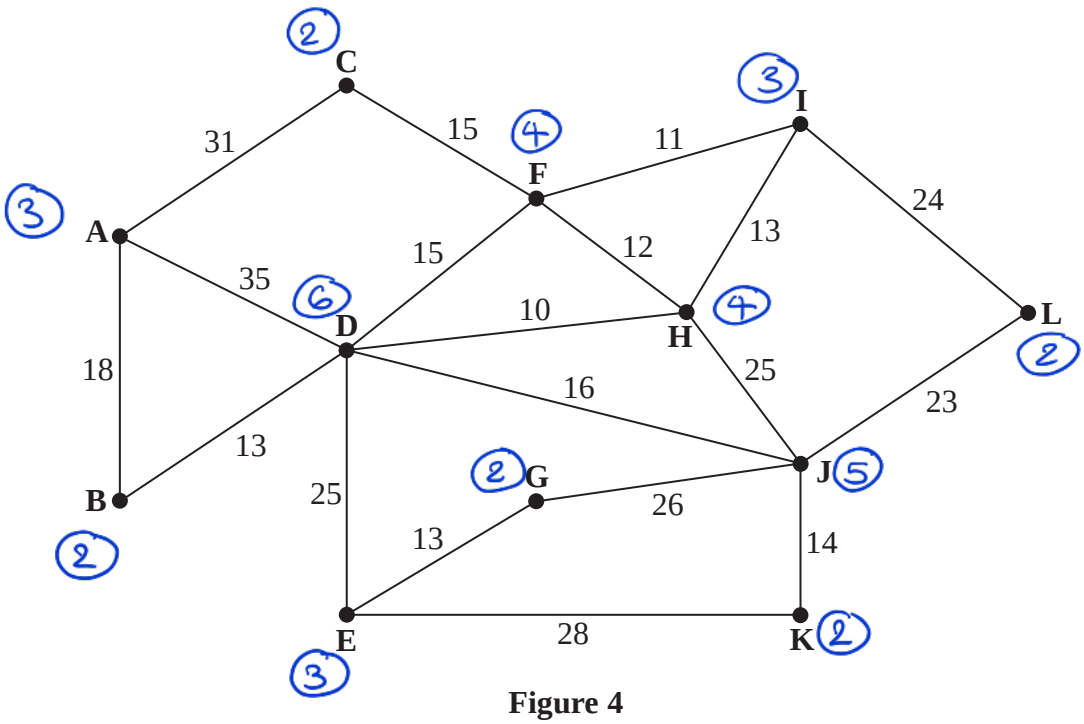


Figure 4

[*The total weight of the network is 367 metres*]

Figure 4 represents a network of water pipes. The number on each arc represents the length, in metres, of that water pipe.

A robot will travel along each pipe to check that the pipe is in good repair.

The robot will travel along each pipe at least once. It will start and finish at A and the total distance travelled must be minimised.

(a) Use the route inspection algorithm to find the pipes that will need to be traversed twice. You must make your method and working clear.

(5)

(b) Write down the length of a shortest inspection route.

(1)

A new pipe, IJ, of length 35 m is added to the network. This pipe must now be included in a new minimum inspection route starting and finishing at A.

(c) Determine if the addition of this pipe will increase or decrease the distance the robot must travel. You must give a reason for your answer.

(2)

(Total 8 marks)

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4.

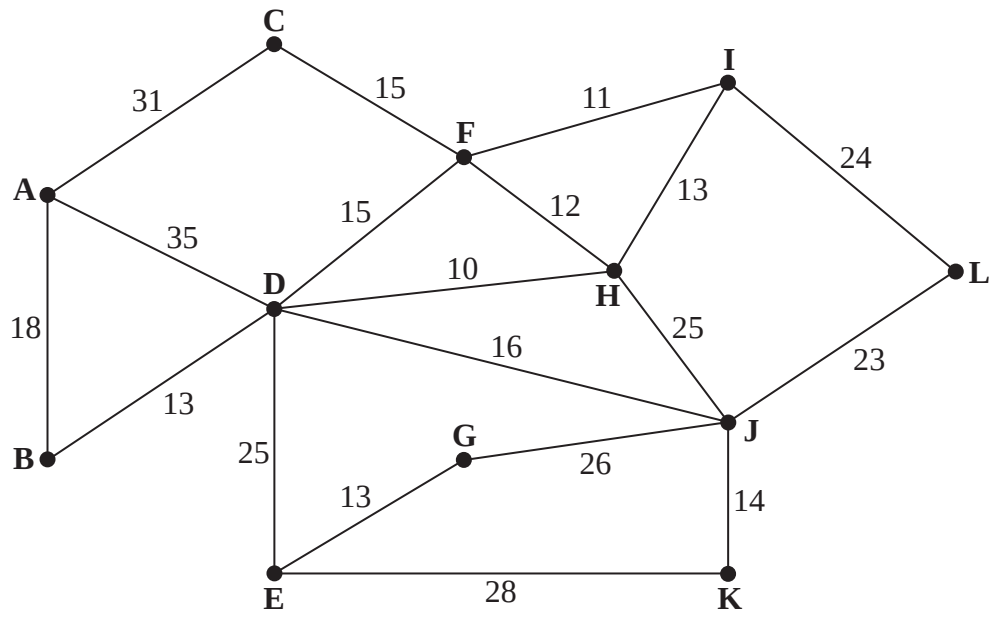


Figure 4

[*The total weight of the network is 367 metres*]

a) Odd nodes : A , E , I , J

$\overset{(BD)}{AE} + \overset{(H)}{IJ}$: (18 + 13 + 25) + (13 + 25) = 94

$\overset{(BDH)}{AI} + \overset{(G)}{EJ}$: (18 + 13 + 10 + 13) + (13 + 26) = 93 * shortest route

$\overset{(BD)}{AJ} + \overset{(DH)}{EI}$: (18 + 13 + 16) + (25 + 10 + 13) = 95

least weight = 93 ∴ repeat AB , BD , DH , HI , EG , GJ

b) 367 + 93 = 460 m

c) odd nodes: A , E

AB , BD and DE are repeated

new length = 367 + 35 + 18 + 13 + 25

= 458 m

∴ distance is decreased

(458 < 460)

5.

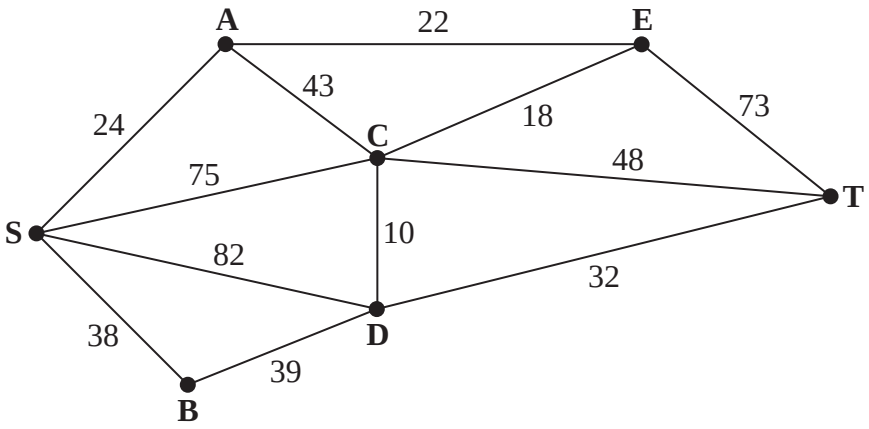


Figure 5

Figure 5 represents a network of roads. The number on each arc represents the length, in km, of the corresponding road.

(a) Use Dijkstra's algorithm to find the shortest route from S to T. State your route and its length.

(6)

The road represented by arc CE is now closed for repairs.

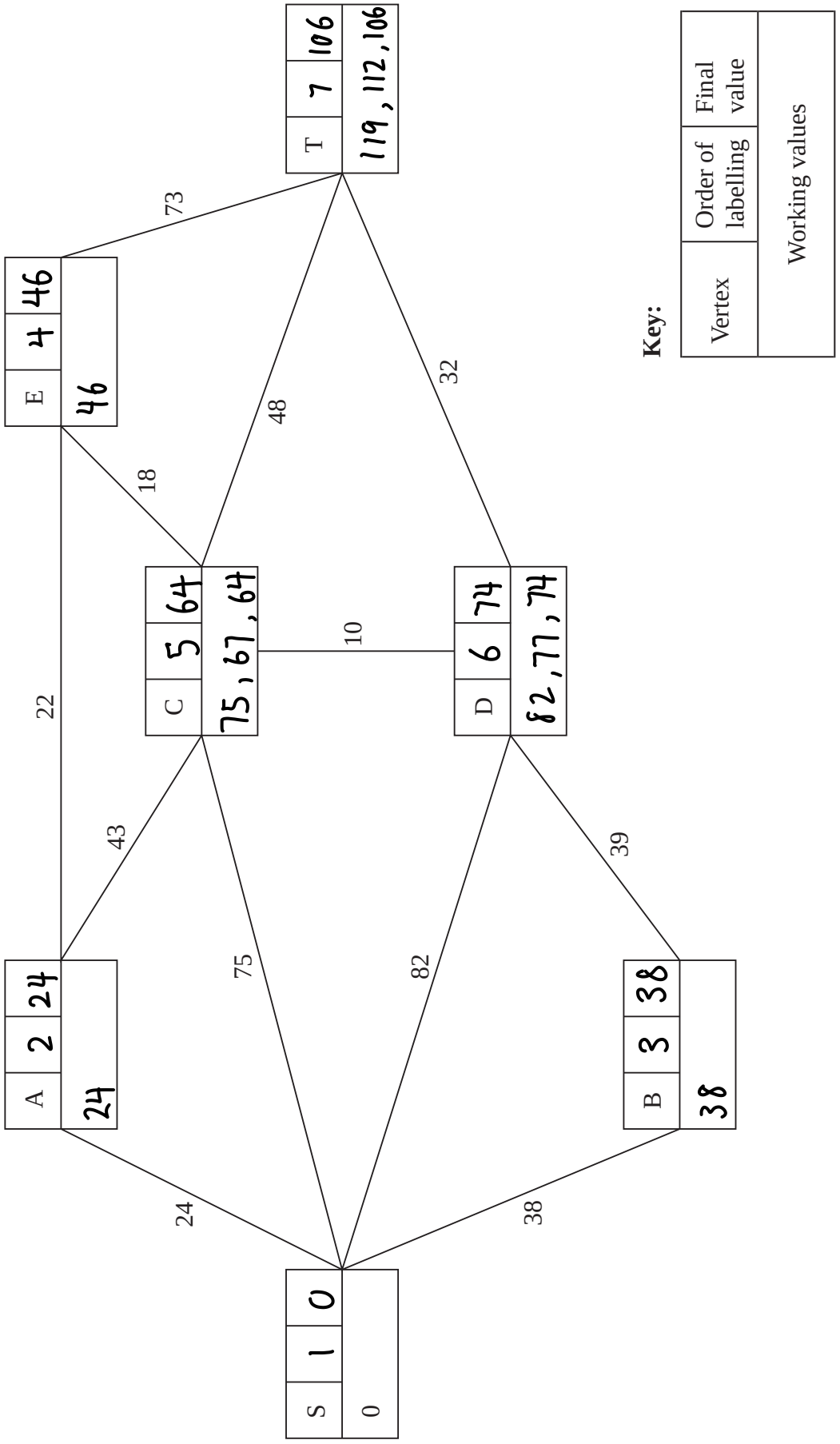
(b) Find **two** shortest routes from S to T that do not include arc CE. State the length of these routes.

(3)

(Total 9 marks)

Leave blank

5. (a)



Question 5 continued

Shortest route from S to T: S - A - E - C - D - T

Length of shortest route from S to T: 106 km

(b)

First shortest route from S to T **excluding arc CE**: S - A - C - D - T

Second shortest route from S to T **excluding arc CE**: S - B - D - T

Length of shortest route from S to T **excluding arc CE**: 109 km

(Total 9 marks)

Q5

Leave blank

6. A linear programming problem in x and y is described as follows.

Minimise $C = 2x + 5y$

subject to

$$x + y \geqslant 500$$
$$5x + 4y \geqslant 4000$$
$$y \leqslant 2x$$
$$y \geqslant x - 250$$
$$x, y \geqslant 0$$

(a) Add lines and shading to Diagram 1 in the answer book to represent these constraints. Hence determine the feasible region and label it R.

(4)

(b) Use **point testing** to determine the exact coordinates of the optimal point, P. You must show your working.

(6)

The first constraint is changed to $x + y \geqslant k$ for some value of k.

(c) Determine the greatest value of k for which P is still the optimal point.

(2)

(Total 12 marks)

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6. $x = 0$

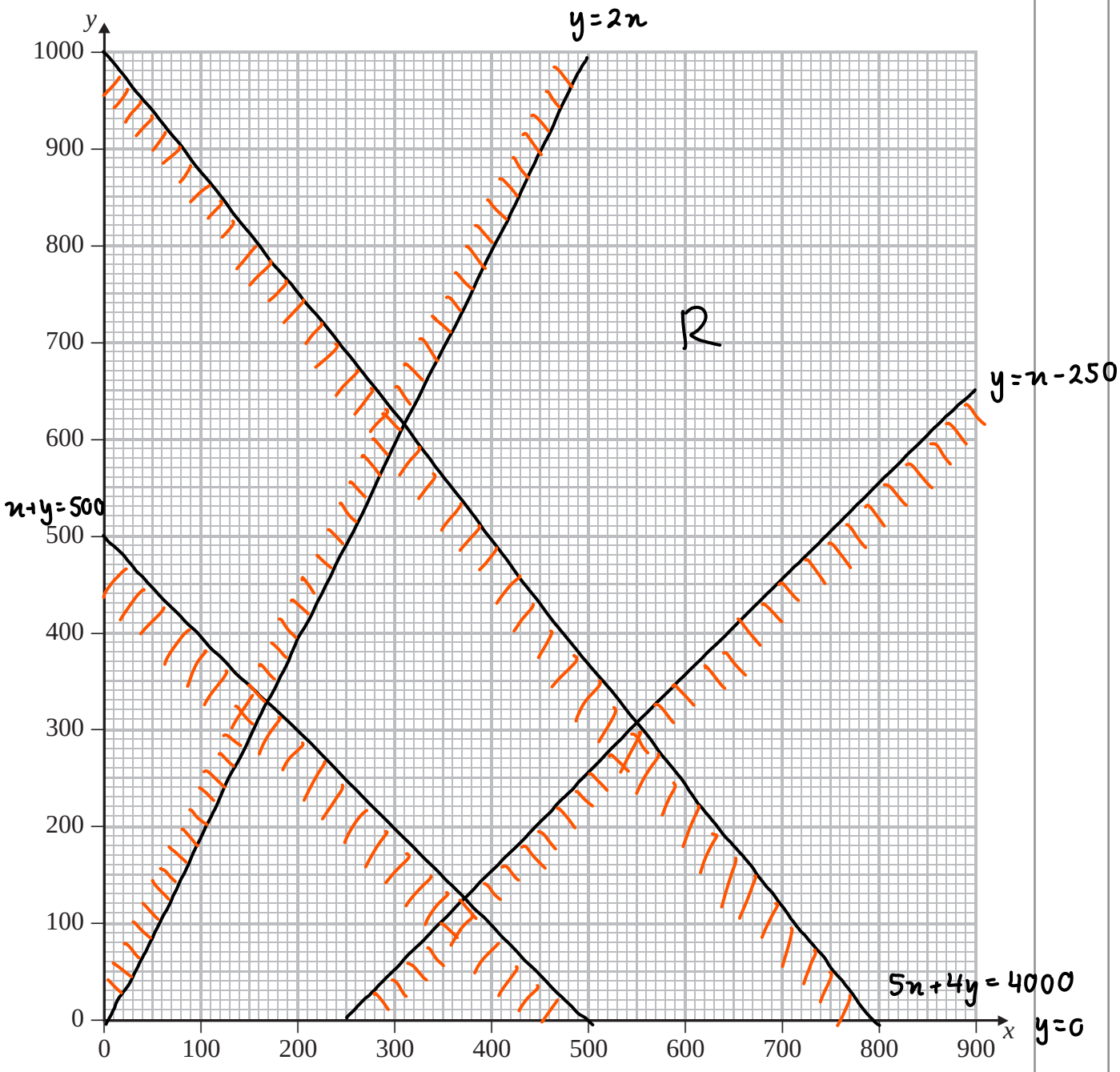


Diagram 1

b) $y = 2x$

$5x + 4y = 4000$

$5x + 4(2x) = 4000$

$x = \frac{4000}{13}$

$y = \frac{8000}{3}$

C: $2\left(\frac{4000}{13}\right) + 5\left(\frac{8000}{13}\right) = \frac{48000}{13}$

≈ 3692.31

P 4 3 0 1 3 A 0 1 2 1 6

Leave blank

Question 6 continued

$y = x - 250$

$5x + 4y = 4000$

$5x + 4(x - 250) = 4000$

$9x = 5000$

$x = \frac{5000}{9}$

$y = \frac{2750}{9}$

C : $2\left(\frac{5000}{9}\right) + 5\left(\frac{2750}{9}\right) = \frac{23750}{9}$

≈ 2638.89

$\therefore$ P : $\left(\frac{5000}{9}, \frac{2750}{9}\right)$

d) At P, $x + y = \frac{7750}{9}$

max value of K : $\frac{7750}{9}$

Q6

(Total 12 marks)

7.

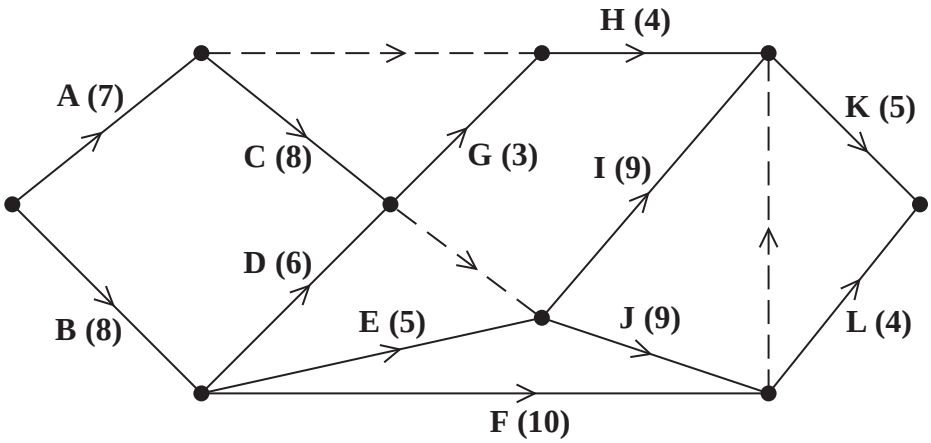


Figure 6

A project is modelled by the activity network shown in Figure 6. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, to complete the activity. Each activity requires one worker. The project is to be completed in the shortest possible time.

(a) Complete Diagram 1 in the answer book to show the early event times and late event times.

(4)

(b) Draw a cascade (Gantt) chart for this project on Grid 1 in the answer book.

(4)

(c) Use your cascade chart to determine a lower bound for the number of workers needed to complete the project in the shortest possible time. You must make specific reference to times and activities.

(2)

The project is to be completed in the minimum time using as few workers as possible.

(d) Schedule the activities, using Grid 2 in the answer book.

(4)

(Total 14 marks)

Leave blank

7. (a)

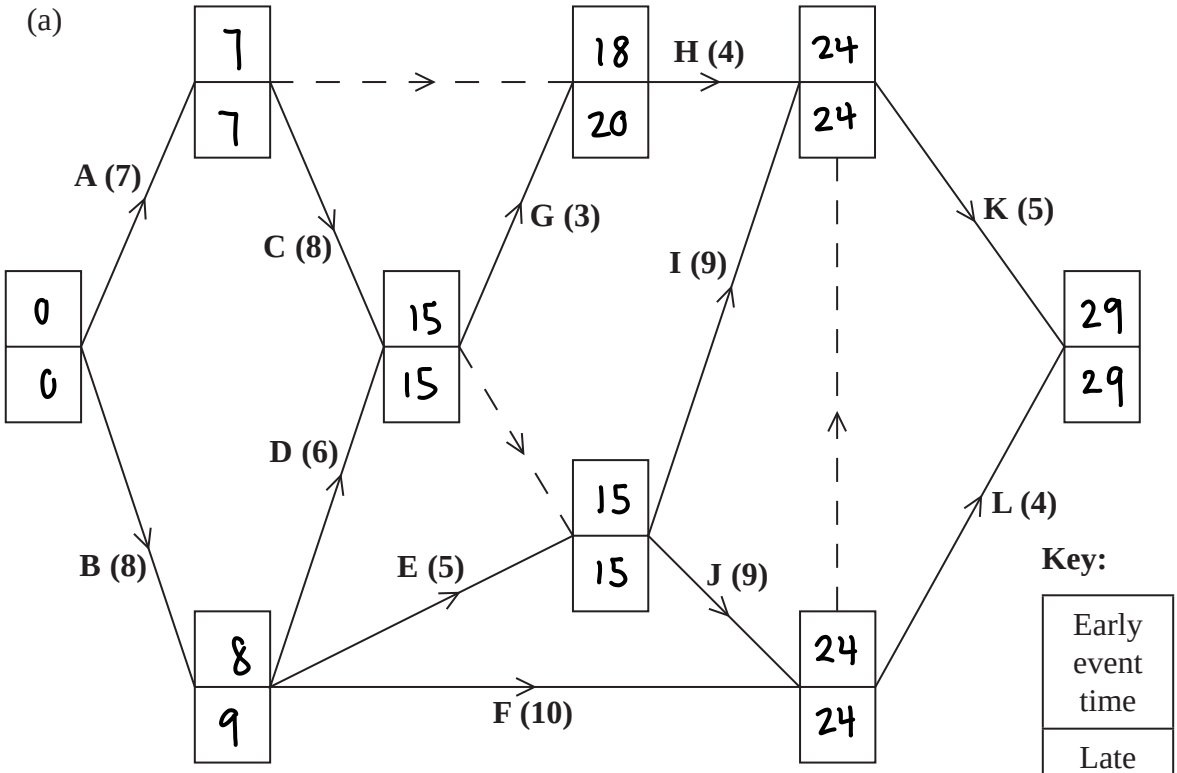
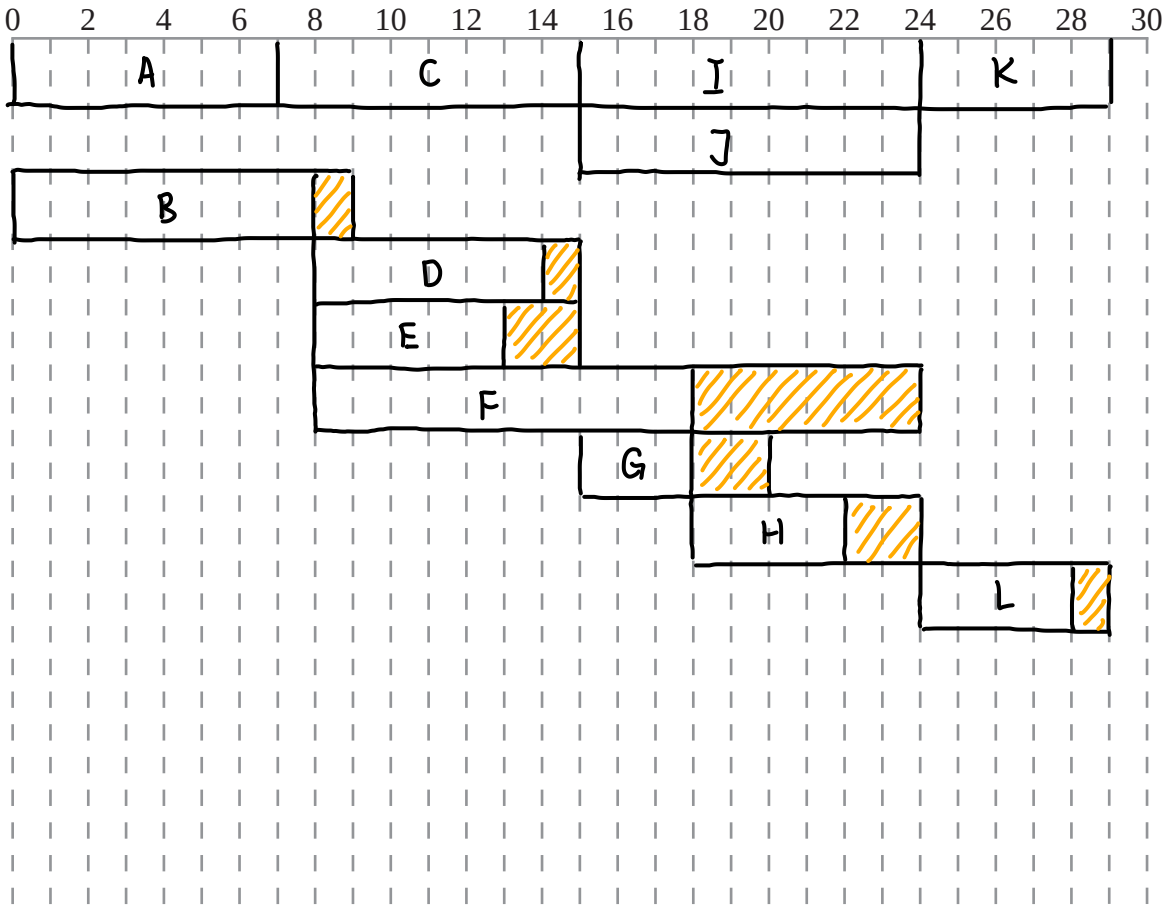


Diagram 1

(b)



Grid 1

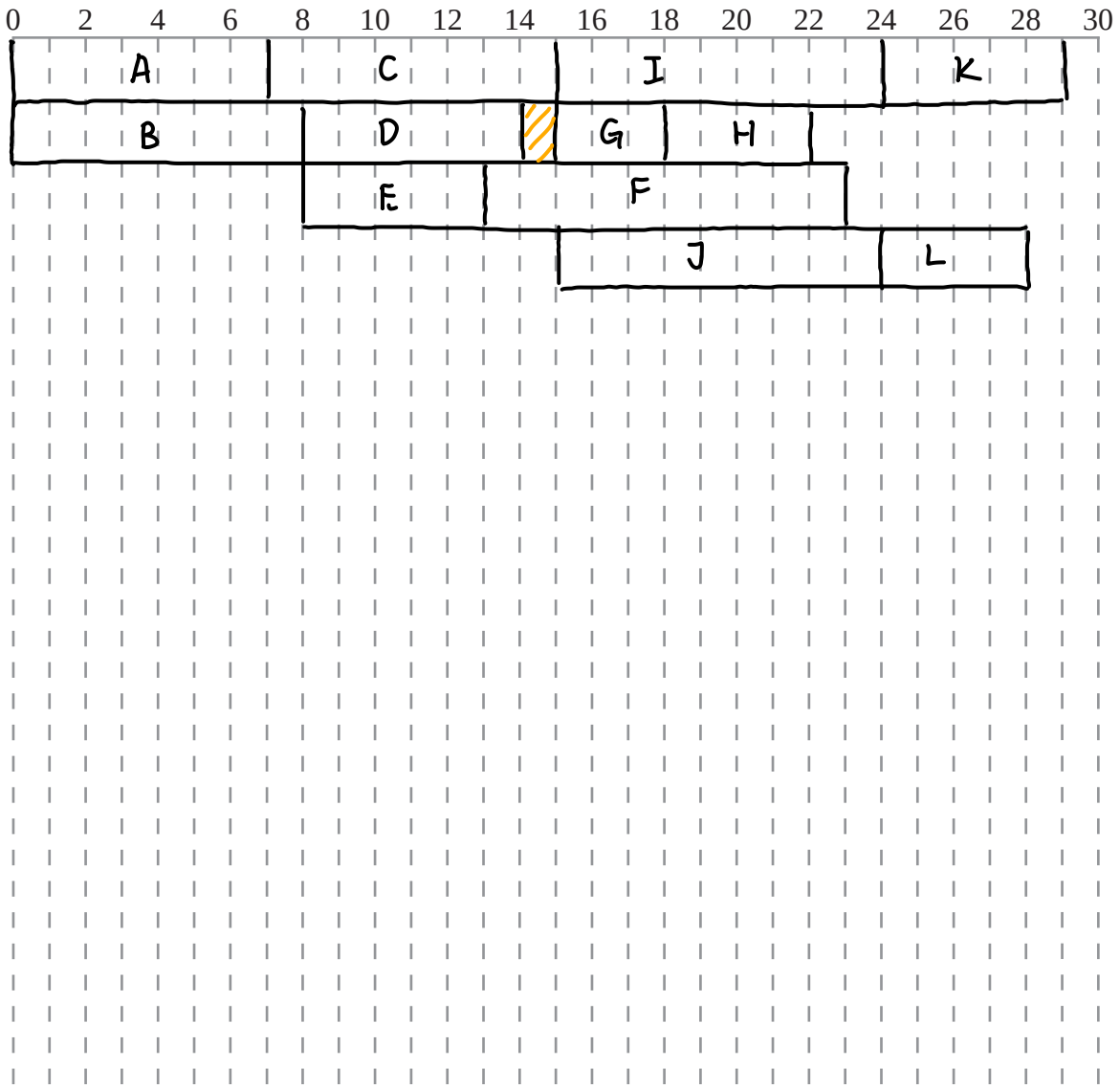
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Question 7 continued

(c)

4 workers, working on activities I, J, F and G from time 17 to 18

(d)



Grid 2

Q7

(Total 14 marks)

8. A charity produces mixed packs of posters and flyers to send out to sponsors.

Pack A contains 40 posters and 20 flyers.

Pack B contains 30 posters and 50 flyers.

The charity must send out at least 15000 flyers.

The charity wants between 40% and 60% of the total packs produced to be Pack As.

Posters cost 15p each and flyers cost 3p each.

The charity wishes to minimise its costs.

Let x represent the number of Pack As produced, and y represent the number of Pack Bs produced.

Formulate this as a linear programming problem, stating the objective and listing the constraints as simplified inequalities with integer coefficients.
You should **not** attempt to solve the problem.

(Total 6 marks)

TOTAL FOR PAPER: 75 MARKS

END

Leave blank

8.

Objective : Minimise $C = [15(40) + 3(20)]x + [15(30) + 3(50)]y$

$= 660x + 600y$

Subject to : $20x + 50y \geq 15000 \Rightarrow 2x + 5y \geq 1500$

$\frac{2}{5}(x+y) \leq x \leq \frac{3}{5}(x+y) \Rightarrow \frac{2}{5}(x+y) \leq x$

$2y \leq 3x$

$\frac{3}{5}(x+y) \geq x$

$3y \geq 2x$

$x \geq 0$

$y \geq 0$

Q8

(Total 6 marks)

TOTAL FOR PAPER: 75 MARKS

END