GradeMax

D1 S14 (R)

1. Mathematics · 2014 · May/Jun · D1 · QP

31 10 38 45 19 47 35 28 12

(a) Use the first-fit bin packing algorithm to determine how the numbers listed above can be packed into bins of size 60

(3)

(b) Carry out a quick sort to produce a list of the numbers in **descending** order. You should show the result of each pass and identify your pivots clearly.

(4)

(c) Use the first-fit decreasing bin packing algorithm to determine how the numbers listed can be packed into bins of size 60

(2)

(d) Determine whether the number of bins used in (c) is optimal. Give a reason for your answer.

(2)

(11 marks)

Bin 1: 31, 10, 19 Bin 2: 38, 12 Bin 3: 45

Bin 4: 47 Bin 5: 35 Bin 6: 28

b) 31 10 38 45 19 47 35 28 12

31 38 45 47 35 28 19 10 12

47 31 38 45 35 28 19 12 10

47 45 31 38 35 28 19 12 10

47 45 38 35 31 28 19 12 10

47 45 38 35 31 28 19 12 10

47 45 38 35 31 28 19 12 10

c) Bin 1: 47, 12 Bin 2: 45, 10 Bin 3: 38, 19

Bin 4: 35 Bin 5: 31, 28

d) lower bound $= \frac{265}{60} = 4.4 \therefore 5$ bins

$\therefore$ c) is an optimal solution.

2.

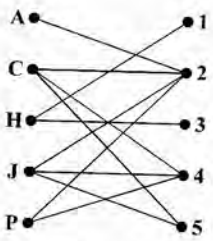


Figure 1

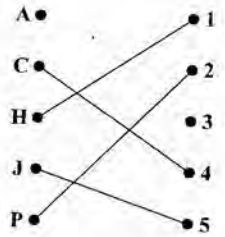


Figure 2

Figure 1 shows the possible allocations of five employees, Ali (A), Campbell (C), Hugo (H), Janelle (J) and Polly (P), to five tasks 1, 2, 3, 4 and 5.

(a) Explain why it is not possible to find a complete matching.

(2)

It is decided that one of the employees should be trained so that a complete matching becomes possible. There are only enough funds for one employee to be trained.

Two employees volunteer to undergo training. Janelle can be trained to do task 1 **or** Hugo can be trained to do task 5.

(b) Decide which employee, Janelle or Hugo, should undergo training. Give a reason for your answer.

(2)

You may now assume that the employee you identified in (b) has successfully undergone training.

Figure 2 shows an initial matching.

(c) Starting from the given initial matching, use the maximum matching algorithm to find a complete matching. You should list the alternating path that you use, and state the complete matching.

(3)

(Total 7 marks)

a) H is the only employee who can do tasks 1 and 3

b) Janelle, Hugo could now do 3 and J-1

c) A-2 = P-4 = C-5 = J-1 = H-3

cs A=2-P=4-C=5-J=1-H=3

Complete

A=2 C=5 H=3 J=1 P=4

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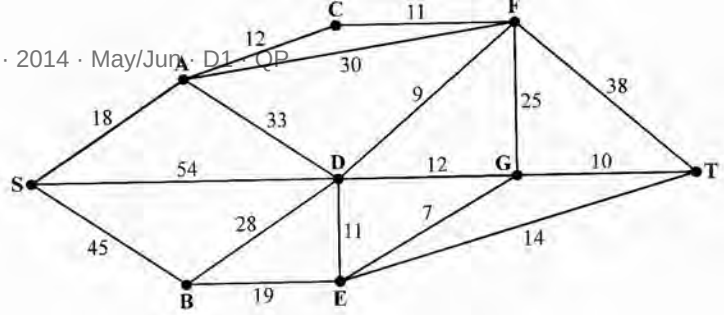


Figure 3

Figure 3 represents a network of roads. The number on each arc represents the time taken, in minutes, to traverse each road.

(a) Use Dijkstra's algorithm to find the quickest route from S to T. State your quickest route and the time taken.

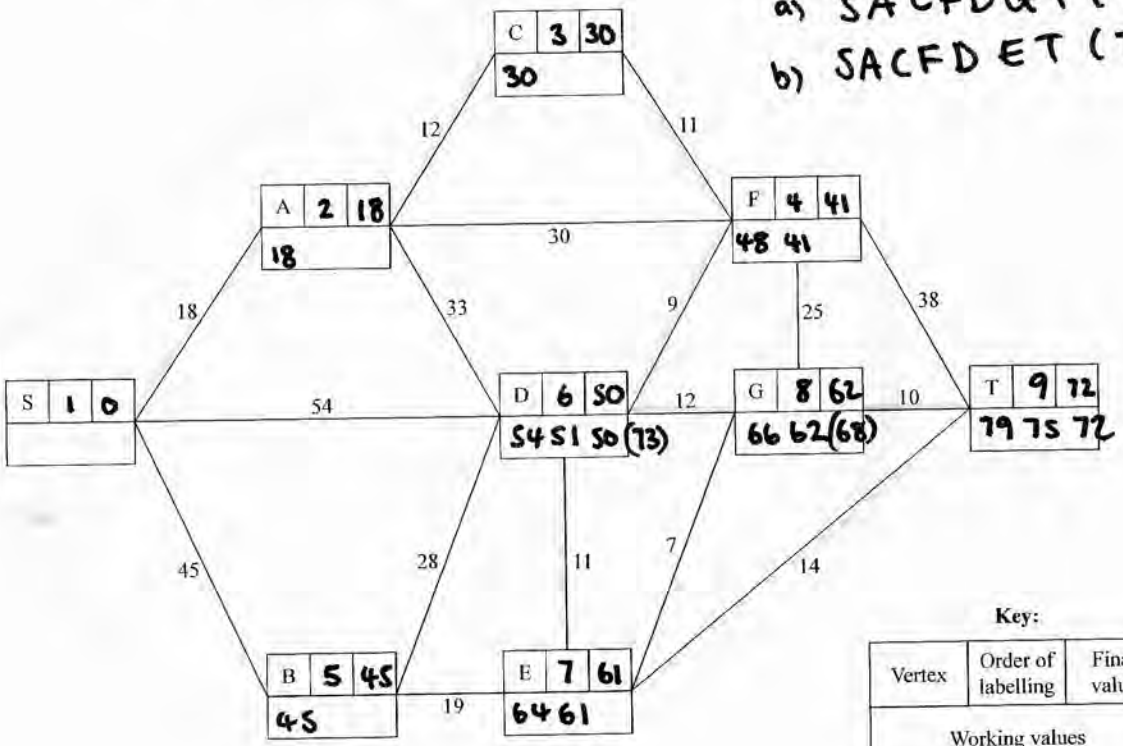
(6)

It is now necessary to include E in the route.

(b) Determine the effect that this will have on the time taken for the journey. You must state your new quickest route and the time it takes.

(3)

a) SACFDGT (72)
b) SACFDET (75)



Key:

Vertex	Order of labelling	Final value
Working values		

4.

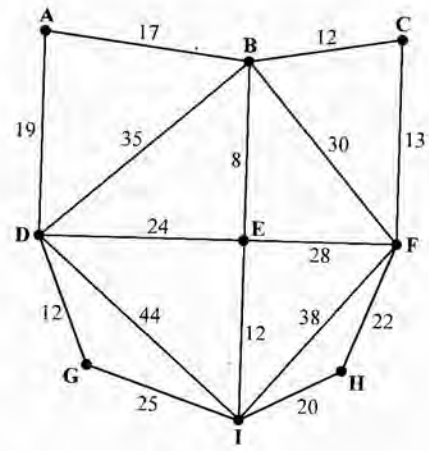


Figure 4

[The total weight of the network is 359 cm]

Figure 4 represents the network of sensor wires used in a medical scanner. The number on each arc represents the length, in cm, of that section of wire.

After production, each scanner is tested.

A machine will be programmed to inspect each section of wire.

It will travel along each arc of the network at least once, starting and finishing at A. Its route must be of minimum length.

(a) Use the route inspection algorithm to find the length of a shortest inspection route. You must make your method and working clear.

(5)

The machine will inspect 15 cm of wire per second.

(b) Calculate the total time taken, in seconds, to test 120 scanners.

(2)

It is now possible for the machine to start at one vertex and finish at a different vertex. An inspection route of minimum length is still required.

(c) Explain why the machine should be programmed to start at a vertex with odd degree.

(2)

Due to constraints at the factory, only B or D can be chosen as the starting point and there will also be a 2 second pause between tests.

(d) Determine the new minimum total time now taken to test 120 scanners. You must state which vertex you are starting from and make your calculations clear.

(4)

(Total 13 marks)

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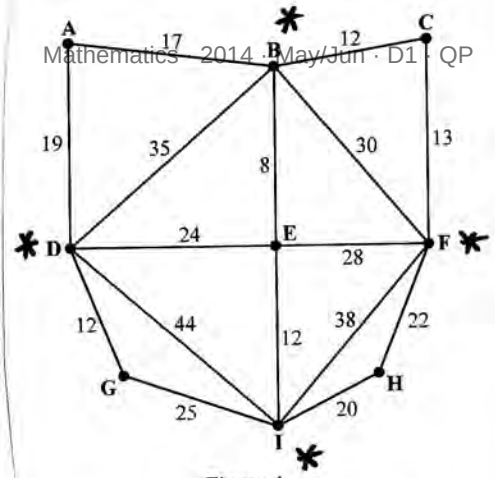


Figure 4

[The total weight of the network is 359 cm]

BD 32 FI 38 ⇒ 70 repeat
E

BF 25 DI 37 ⇒ 62 * BC, CF, DG, GI
C G +62
= 421
BI 20 FD 52 ⇒ 72 2
E E

b) $\frac{421 \times 120}{15} = 3368$ sec.

c) Problem can now be solved with a semi-Eulerian graph, where the start and end nodes are the odd pair.
as the route passes through a node it adds two to its order ∴ the start/end node must be odd.

d) Start from D, end at F
repeat BE, EI +20 = 379cm
119 pauses @ 2sec = 238

$\frac{379}{15} \times 120 + 238 = 3270$ sec
2

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5. A linear programming problem in x and y is described as follows.

Maximise $P = 2x + 3y$

subject to

$$x \geqslant 25$$
$$y \geqslant 25$$
$$7x + 8y \leqslant 840$$
$$4y \leqslant 5x$$
$$5y \geqslant 3x$$
$$x, y \geqslant 0$$

(a) Add lines and shading to Diagram 1 in the answer book to represent these constraints. Hence determine the feasible region and label it R.

(4)

(b) Use the objective line method to find the optimal vertex, V, of the feasible region. You must clearly draw and label your objective line and the vertex V.

(3)

(c) Calculate the exact coordinates of vertex V.

(2)

Given that an integer solution is required,

(d) determine the optimal solution with integer coordinates. You must make your method clear.

(2)

(Total 11 marks)

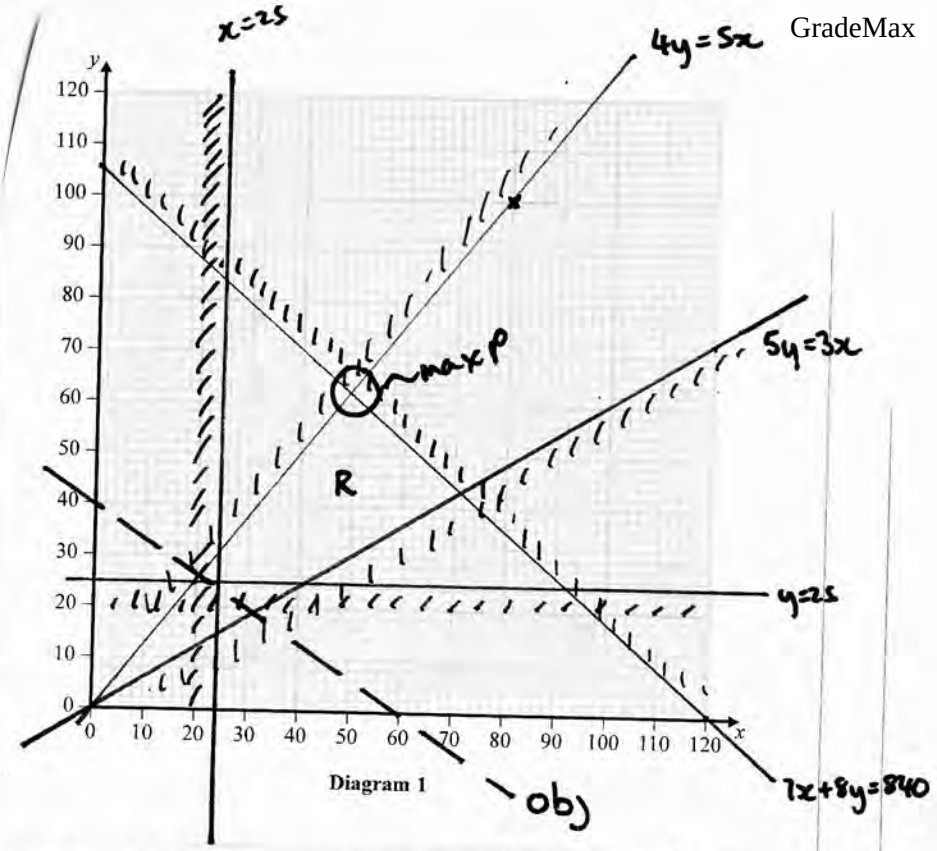


Diagram 1

$P = 2x + 3y$ $\quad 2x + 3y = 120$ $\quad y = 40$, $x = 60$

$4y = 5x$ $\quad 8y = 10x$ $\quad y \leqslant \frac{5}{4}x$

$7x + 8y = 840$ $\quad 17x = 840$ $\quad x = \frac{840}{17}$ $\quad y = \frac{1050}{17}$

(49,62) (50,62) (49·4, 61·8) (49,61) (50,61)

	$4y \leqslant 5x$	$7x + 8y \leqslant 840$	$P = 2x + 3y$
(50,62)	62·5 ✓	846 ✗	—
(49,62)	61·25 ✗		
(50,61)	62·5 ✓	838 ✓	€283
(49,61)	61·25 ✓	831 ✓	€281

$\therefore x = 50, y = 61$

$\max P = €283$

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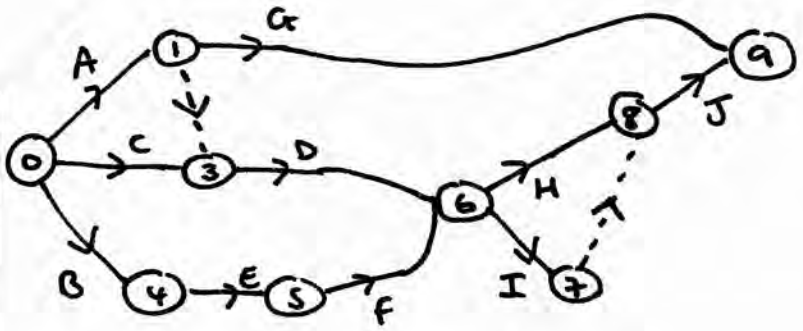
6. (i) Draw the activity network described in the precedence table below, using activity on arc and the minimum number of dummies.

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Activity	Immediately preceding activities
A	–
B	–
C	–
D	*A, C*
E	*B*
F	*E*
G	*A*
H	*D, F*
I	*D, F*
J	*H, I*

(ii) Explain why each of your dummies is necessary.

(Total 7 marks)



①-③ G depends on A but D depends on A and C

⑦-⑧ No two activities may share the same start and end event.

8. A manufacturer of frozen yoghurt is going to exhibit at a trade fair. He will take two yoghurt, Banana Blast and Strawberry Scream.

He will take a total of at least 1000 litres of yoghurt.

He wants at least 25% of the yoghurt to be Banana Blast. He also wants there to be at most half as much Banana Blast as Strawberry Scream.

Each litre of Banana Blast costs £3 to produce and each litre of Strawberry Scream costs £2 to produce. The manufacturer wants to minimise his costs.

Let x represent the number of litres of Banana Blast and y represent the number of litres of Strawberry Scream.

Formulate this as a linear programming problem, stating the objective and listing the constraints as simplified inequalities with integer coefficients.

You should **not** attempt to solve the problem.

(Total 6 marks)

$x + y \geq 1000$

$25\%(x+y) \leq x$ $\quad y \geq 2x$

$C = 3x + 2y$ $\quad 0.25x + 0.25y \leq x$

$0.25y \leq 0.75x$

$2y \leq 6x$

∴ objective is to min costs C(£)

$C = 3x + 2y$

Subject to

$x + y \geq 1000$ $\quad y \geq 2x$ $\quad y \leq 3x$

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7.

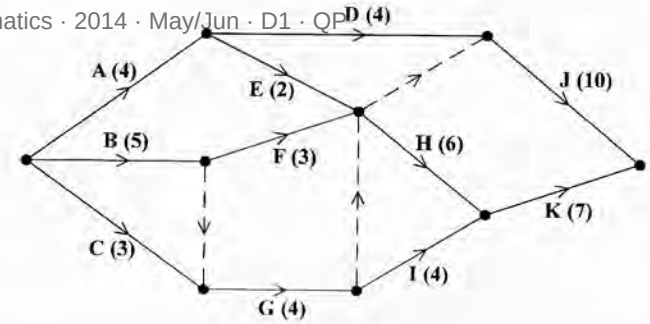


Figure 5

A project is modelled by the activity network shown in Figure 5. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, to complete the activity. Each activity requires one worker. The project is to be completed in the shortest possible time.

(a) Complete Diagram 1 in the answer book to show the early event times and late event times.

(4)

(b) Calculate the total float for activity D. You must make the numbers you use in your calculation clear.

(2)

(c) Calculate a lower bound for the number of workers needed to complete the project in the minimum time. You must show your working.

(2)

The project is to be completed in the minimum time using as few workers as possible.

(d) Schedule the activities using Grid 1 in the answer book.

(3)

(Total 11 marks)

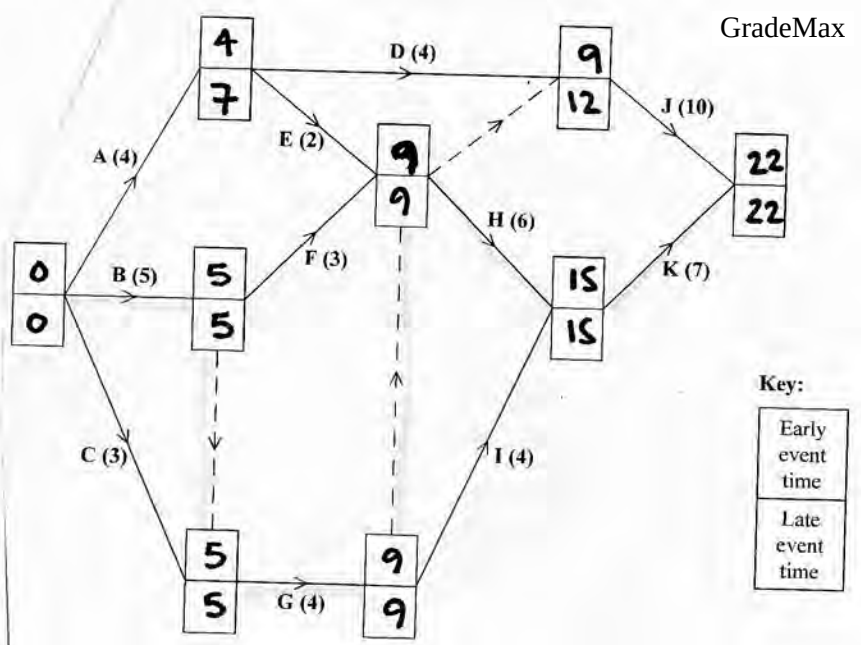


Diagram 1

(b)

float on D = 12 − 4 − 4 = 4

LB = $\frac{52}{22}$ = 2.4 → 3 workers

