GradeMax

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WFM02/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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1. Solve the equation

$$z^5 - 32i = 0$$

giving each answer in the form $re^{i\theta}$ where $0 < \theta < 2\pi$

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Question 1 continued

Q1

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2. Use algebra to determine the set of values of x for which

$$\frac{x}{2-x} \leqslant \frac{x+3}{x}$$

(*Solutions relying entirely on graphical methods are not acceptable*.)

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Question 2 continued

Q2

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3. A transformation maps points from the z-plane, where $z = x + iy$, to the w-plane, where $w = u + iv$. The transformation is given by

$$w = \frac{(2+i)z + 4}{z - i} \qquad z \neq i$$

The transformation maps the imaginary axis in the z-plane onto the line l in the w-plane.

Determine a Cartesian equation of l, giving your answer in the form $au + bv + c = 0$ where a, b and c are integers to be found.

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Question 3 continued

Q3

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4. (a) Determine the general solution of the differential equation

$$(x+1)\frac{\mathrm{d}y}{\mathrm{d}x} - xy = \mathrm{e}^{3x} \qquad x > -1$$

giving your answer in the form $y = \mathrm{f}(x)$.

(7)

(b) Determine the particular solution of the differential equation for which $y = 5$ when $x = 0$

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Question 4 continued

Q4

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5. Given that $y = \tan^2 x$

(a) show that

$$\frac{\mathrm{d}^3 y}{\mathrm{d}x^3} = 8 \tan x \sec^2 x \left(p \sec^2 x + q\right)$$

where p and q are integers to be determined.

(5)

(b) Hence determine the Taylor series expansion about $\frac{\pi}{3}$ of $\tan^2 x$ in ascending powers of $\left(x - \frac{\pi}{3}\right)$ up to and including the term in $\left(x - \frac{\pi}{3}\right)^3$, giving each coefficient in simplest form.

(3)

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Question 5 continued

Q5

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6. The complex number z on an Argand diagram is represented by the point P where

$$|z + 1 - 13i| = 3|z - 7 - 5i|$$

Given that the locus of P is a circle,

(a) determine the centre and radius of this circle.

(5)

The complex number w, on the same Argand diagram, is represented by the point Q, where

$$\arg(w - 8 - 6i) = -\frac{3\pi}{4}$$

Given that the locus of P intersects the locus of Q at the point R,

(b) determine the complex number representing R.

(4)

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Question 6 continued

Q6

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7. (a) Show that the transformation $x = t^2$ transforms the differential equation

$$4x\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2\left(1 + 2\sqrt{x}\right)\frac{\mathrm{d}y}{\mathrm{d}x} - 15y = 15x \qquad \text{(I)}$$

into the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} + 2\frac{\mathrm{d}y}{\mathrm{d}t} - 15y = 15t^2 \qquad \text{(II)}$$

(5)

(b) Solve differential equation (II) to determine y in terms of t.

(5)

(c) Hence determine the general solution of differential equation (I).

(1)

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Question 7 continued

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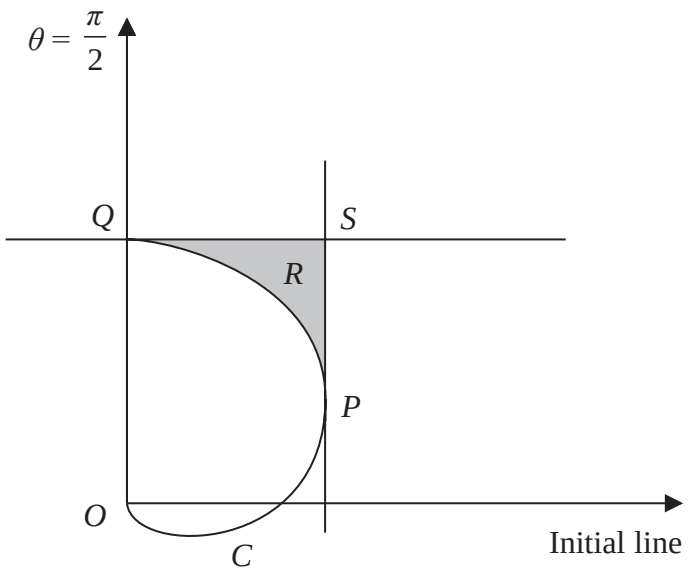


Figure 1

The curve C shown in Figure 1 has polar equation

$$r = 1 + \sin\theta \qquad -\frac{\pi}{2} < \theta \leqslant \frac{\pi}{2}$$

The point P lies on C such that the tangent to C at P is perpendicular to the initial line.

(a) Use calculus to determine the polar coordinates of P.

(5)

The tangent to C at the point Q where $\theta = \frac{\pi}{2}$ is parallel to the initial line.

The tangent to C at Q meets the tangent to C at P at the point S, as shown in Figure 1.

The finite region R, shown shaded in Figure 1, is bounded by the line segments QS, SP and the curve C.

(b) Use algebraic integration to show that the area of R is

$$\frac{1}{32}\left(a\sqrt{3} + b\pi\right)$$

where a and b are integers to be determined.

(6)

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9. (a) Show that

$$n^5 - (n-1)^5 \equiv 5n^4 - 10n^3 + 10n^2 - 5n + 1$$

(2)

(b) Hence, using the method of differences, show that for all integer values of n,

$$\sum_{r=1}^{n} r^4 = \frac{1}{30} n(n+1)(2n+1)(an^2 + bn + c)$$

where a, b and c are integers to be determined.

(7)

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TOTAL FOR PAPER: 75 MARKS

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